Division of Mathematics
Alfred University

Alfred, NY 14802

Instructions:

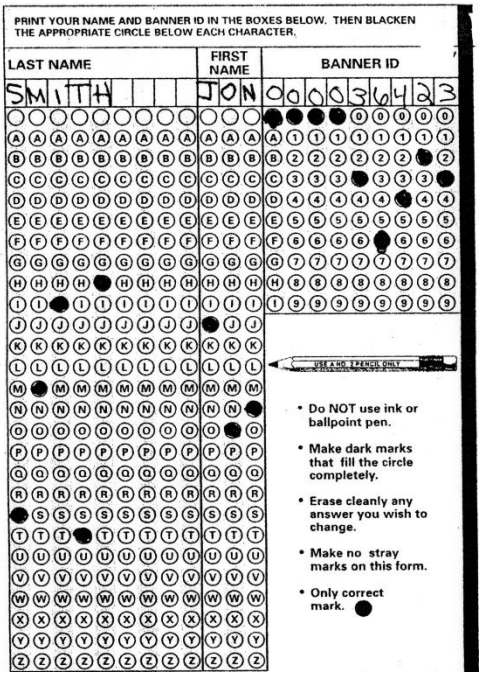
1. This competition will last seventy-five minutes – from 10:05 to 11:20.

2. The use of calculators is not permitted.

3. There are thirty questions. Mark your answers on the computer answer sheet. Use a #2 pencil only. You may use this question booklet for scratch work. You may keep this booklet.

4. The last two pages contain puzzles for your enjoyment; these are not part of the competition.

5. Put your name on the Scantron. Your last name must start in the left column of the last name section. You do not need to put blanks at the end of your name. Put your nine-digit student number in the Banner ID on the Scantron sheet. You **must** include the four zeros at the beginning of this number. If your name and number are not on the sheet your answers will not be graded.
   Example:



1) There are 40 big and small barrels. The big barrels each contain 5 gallons of gas and the small ones each contain 3 gallons of gas. The big barrels contain 24 gallons in total more than the small ones. How many big barrels are there?

A) 16

B) 18

C) 20

D) 22

E) 24

2) If $+?=5, ?+@=3, and $+@=-1, what does $ equal?

A) -1/2

B) 1/2

C) 1

D) 3/2

E) 2

3) A customer has two pennies, a nickel, a dime and a quarter to make a purchase. The customer always uses the right combination of coins so that they get back the minimum number of coins in change. In turn the clerk always gives back the minimum number of coins possible. What is the maximum number of coins the customer might receive in change for making a purchase from 1¢ to 42¢?

A) 2

B) 3

C) 4

D) 5

E) 6

4) What is the largest area that a triangle inscribed in a semi-circle, with its hypotenuse the diameter d, can have?

A) $\frac{\pi d^2}{4}$

B) 12

C) $\frac{d^2}{8}$

D) $\frac{d^2}{4}$

E) $\frac{d^2}{4\pi}$

5) A town's population is increased by 1200 people, and then this new population is decreased by 11%. The town now has 32 fewer people than it did before the 1200 increase. What was the original population?

A) 1200

B) 9,968

C) 10,000

D) 11,200

E) 12,000

6) What is the maximum number of integers that we could choose from the list 1,2,3,4,5,…,2009 such that the difference between any two of the chosen integers is not equal to 8?

A) 990

B) 1000

C) 1002

D) 1006

E) 1008

7) We will conduct an experiment that will take only one minute. In the first six seconds we take two balls put them in a barrel and pull out one ball. In the next six seconds we take three balls put them in the barrel and pull out one ball. In the next six seconds we take four balls put them in the barrel and pull out one ball. This process continues for one minute. How many balls are in the barrel at the end of the minute?

A) 52

B) 53

C) 54

D) 55

E) 56

8) In a certain country's currency the area of the largest coin is twice that of the smallest coin. If four of the largest coins are placed next to each other along a straight line and they span a distance of 10 inches, how long would a chain of four of the smallest coins be?

A) 2.5 inches

B) $\frac{5\sqrt{2}}{2}$ inches

C) 5 inches

D) $5\sqrt{2}$ inches

E) $10\sqrt{2}$ inches

9) Let $f(x)=\frac{1}{2}x^3-\frac{3}{2}x$. What does $f(f(f(f(f(f(f(f(f(f(2)))))))))$ equal?

A) -2

B) -1

C) 0

D) 1

E) 2

10) How many triples of the form (x, y, z) satisfy the system of equations $\begin{cases} x^2+y^2-z^2=1 \\ y^2+z^2-x^2=2 \\ z^2+x^2-y^2=3 \end{cases}$?

A) None

B) 3

C) 6

D) 8

E) Infinitely many

11) Jane deposited some money in an account at Volatile Bank. During the first week, she quadrupled her money then lost $800. During the second week, she tripled her money then lost $2,000. During the third week, she doubled her money then lost $1,400. She then closed the account, which contained $600. Which of the following is true?

A) Jane made $150.

B) Jane lost $150.

C) Jane made $600.

D) Jane lost $600.

E) Jane broke even.

12) The expression $(((7^7)^7)^{7\cdots})^7$ has 2009 exponents, in decimal form the last digit is

A) 1

B) 3

C) 5

D) 7

E) 9

13) Each of the six letters in the cryptarithm correspond to one and only one of the digits 1, 2, 4, 6, 8, and 9. Determine the digit to which T corresponds.

$$\begin{array}{r} T\ W\ O \\ +\ T\ W\ O \\ \hline F\ O\ U\ R \end{array}$$

A) 2

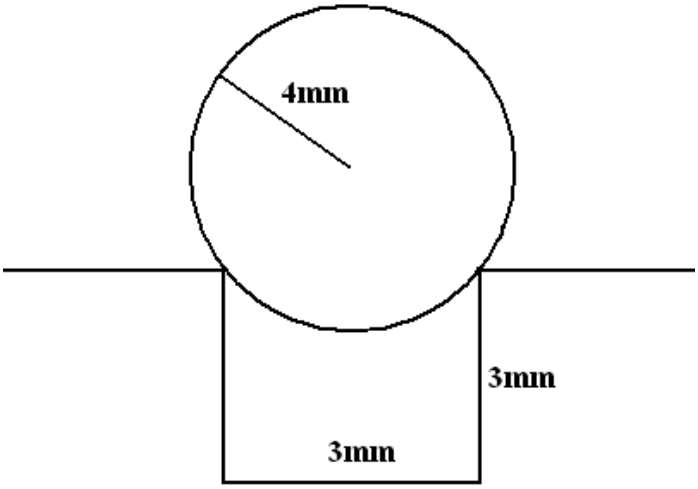
B) 4

C) 6

D) 8

E) 9

14) A circle with radius 4mm is placed into a groove with width and depth 3mm. How far is the center of the circle from the bottom of the groove?



A) 7 mm

B) $7-\frac{\sqrt{26}}{3}$ mm

C) $2\pi-\frac{3}{4}$ mm

D) $2\pi+\frac{4}{3}$ mm

E) $3+\frac{\sqrt{55}}{2}$ mm

15) If an average speed of 60 mph will get you there ½ hour early, and an average speed of 40 mph will get you there ½ hour late, then what speed will get you there on time?

A) 48 mph

B) 49 mph

C) 50 mph

D) 51 mph

E) 52 mph

16) Jeff and Lisa are swimming in a pool with length 30m. Lisa swims 1m/s while Jeff swims 0.6m/s. If they started at the same time from opposite ends of the pool, and they swim back and forth for 10 minutes, how many times would they meet each other?

A) 16

B) 18

C) 20

D) 22

E) 24

17) Of all three digit numbers, none of whose digits is zero, how many will satisfy the property that their digits are in strictly increasing order (reading from left to right)?

A) 56

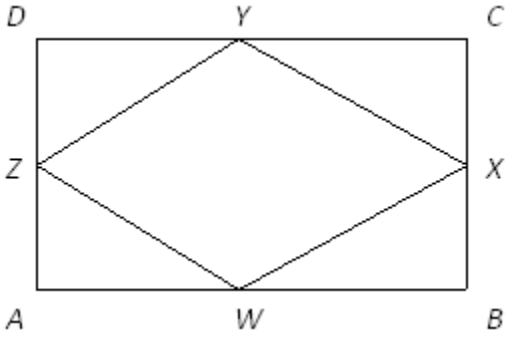
B) 84

C) 120

D) 165

E) 220

18) Rectangle *ABCD* is twice as long as it is high. Parallelogram *WXYZ* is formed by connecting the midpoints of the sides of *ABCD*. Find the ratio of the perimeter of *WXYZ* to that of *ABCD*.



A) $\frac{\sqrt{2}}{3}$

B) $\frac{1}{2}$

C) $\frac{\sqrt{3}}{3}$

D) $\frac{2}{3}$

E) $\frac{\sqrt{5}}{3}$

19) If r and s are numbers such that r + s = 7 and rs = 9, then $(r - s)^2$ =

A) $\sqrt{7}$

B) 3

C) 13

D) 31

E) 63

20) In the accompanying figure the diameter BC of the smaller circle is 4, point C is the center of the larger circle and point B is the midpoint of segment AC. The area of the region between the two circles is what percentage of the area of the larger circle?

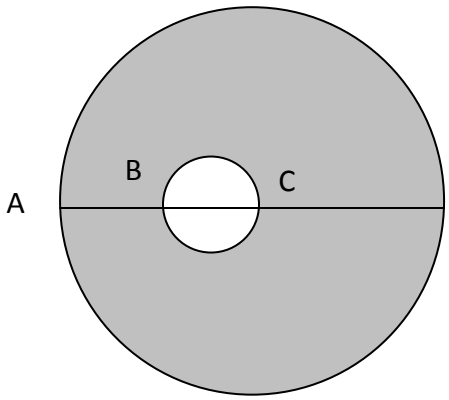
A) 50%

B) 64%

C) 84.5%

D) 87.5%

E) 93.75%



21) A layer of plastic reduces the intensity of the light by 10%. What is the minimum number of layers required to reduce the intensity to 50% (or less) of the original?

A) 5

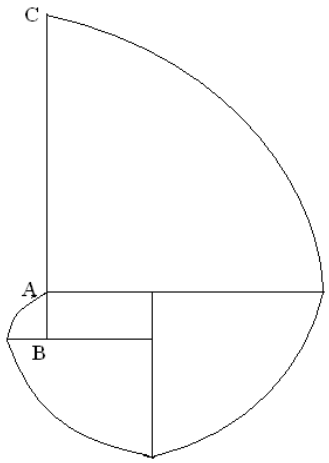
B) 6

C) 7

D) 8

E) 10

22) A goat is tied to corner A of a rectangular building with a rope of length 6m. The width of the building is 2m and the length 1m. If the goat starts at point C and walks around the building clockwise as in the diagram, what is the area that the goat sweeps out?



A) $36\ \pi\ m^2$

B) $15\ \pi\ m^2$

C) $15.5\ \pi\ m^2$

D) $16\ \pi\ m^2$

E) $20\ \pi\ m^2$

23) From the epitaph for the great mathematician Diophantus:
Diophantus passed one sixth of his life in childhood,
one twelfth in youth, and one seventh more as a bachelor;
five years after his marriage, a son was born,
who died four years before his father at half his final age.
Find Diophantus's age at his death.

A) 72 years

B) 76 years

C) 80 years

D) 84 years

E) 88 years

24) A portion of a white square is painted black as follows:
a) The square is divided into four equally sized squares.
b) One of the four squares is painted black.
c) One of the three remaining white squares is divided into four equally sized squares, and the process is repeated.
If this process could be repeated infinitely often, what portion of the square would be painted black?

A) $\frac{1}{3}$

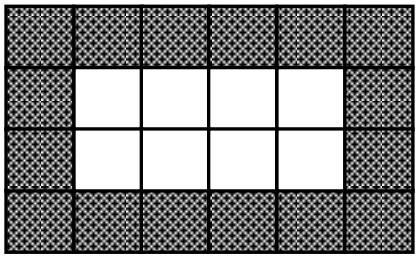
B) $\frac{4}{7}$

C) $\frac{1}{4}$

D) $\frac{2}{3}$

E) $\frac{2}{5}$

25) A company makes rectangular area rugs which are sewn together using 1 foot by 1 foot white and black squares. The rugs are white with a 1 foot black border, as in the example shown.



A customer wants a rug which has the same number of black squares as white. How many different sized rugs can be made with this specification?

A) None

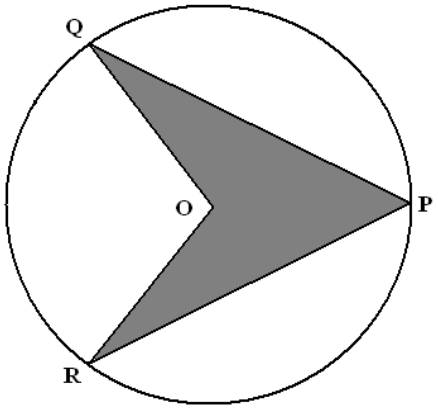
B) 1

C) 2

D) 3

E) Infinitely many

26) Find the area of the shaded figure if the point O is at the center of the circle, the shaded figure is symmetric over the line OP and angle QPR = 60^0.



A) $\frac{\sqrt{3}}{2}OQ^2$

B) $\frac{\sqrt{5}}{2}OQ^2$

C) $\frac{1}{2}OQ^2$

D) $\frac{\sqrt{\pi}}{4}OQ^2$

E) $\frac{\sqrt{3}}{4}OQ$

27) If $x^3 + \frac{1}{x^3} = -2$ and $x^2 + \frac{1}{x^2} = 2$, then $x + \frac{1}{x} =$

A) -3

B) 3

C) -2

D) 2

E) -1

28) If the area of the smaller square is 1 square foot, then the area of the larger circle is

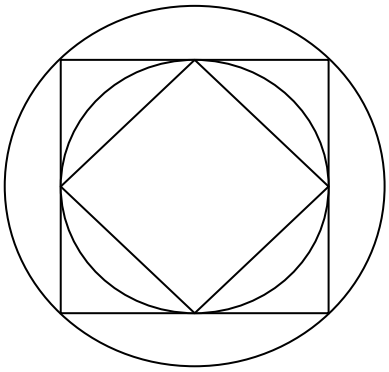
A) 2 sq. ft.

B) π sq. ft.

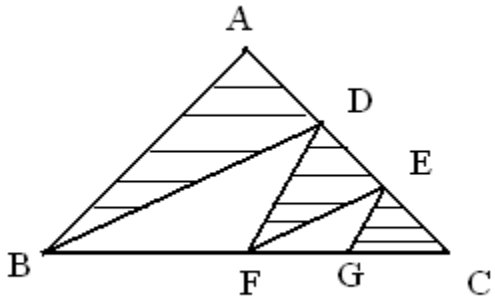
C) 3 sq. ft.

D) 2π sq. ft.

E) 4 sq. ft.



29) Suppose the area of ΔABC is 24 cm^2, AD = DE = EC, BF = FC, and FG = GC as shown in the following graph, the area of the shaded region is



A) 12 cm^2

B) 14 cm^2

C) 16 cm^2

D) 10 cm^2

E) 8 cm^2

30) $1-\frac{1}{1+2}-\frac{1}{1+2+3}-\frac{1}{1+2+3+4}-.....-\frac{1}{1+2+3+...+100}=$

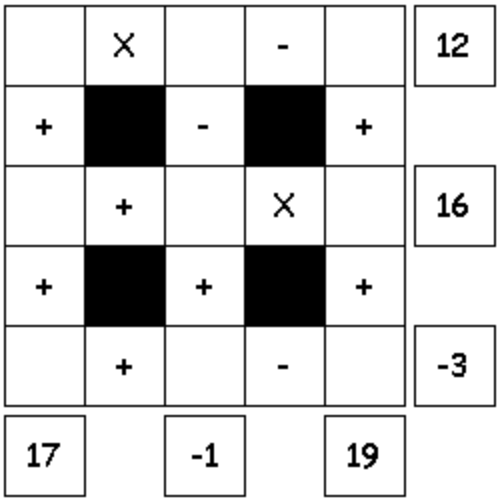
A) $\frac{1}{100}$

B) $\frac{1}{50}$

C) $\frac{3}{100}$

D) $\frac{2}{101}$

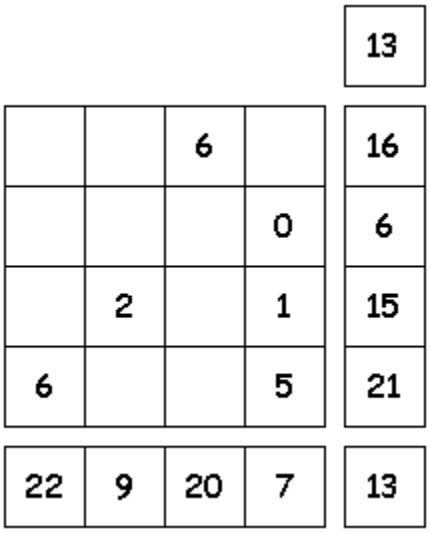
E) $\frac{1}{51}$



Try to fill in the missing numbers.

Use the numbers 1 through 9 to complete the equations.

Each number is only used once.
Each row is a math equation. Each column is a math equation.
Remember that multiplication and division are performed before addition and subtraction.



Try to fill in the missing numbers.

The missing numbers are integers between 0 and 9.
The numbers in each row add up to totals to the right.
The numbers in each column add up to the totals along the bottom.
The diagonal lines also add up the totals to the right.