AU

William Varick Nevins III High School Mathematics Competition
Fall 2008

Division of Mathematics
Alfred University

Alfred, NY 14802

Instructions:

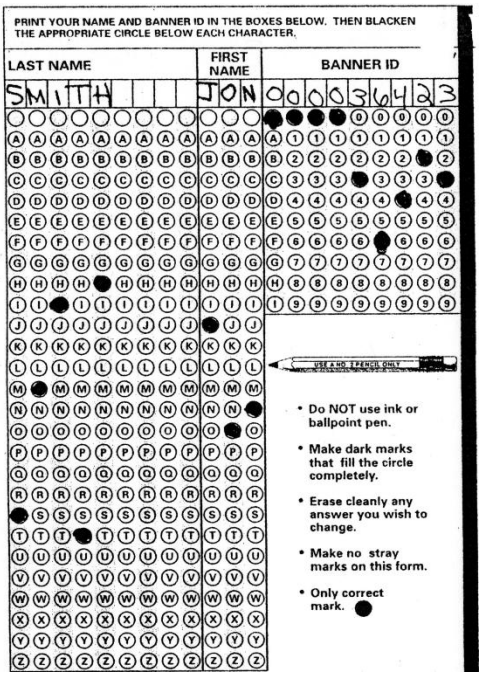
1. This competition will last seventy-five minutes – from 10:05 to 11:20.

2. The use of calculators is not permitted.

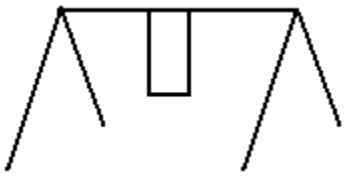
3. There are thirty questions. Mark your answers on the computer answer sheet. Use a #2 pencil only. You may use this question booklet for scratch work. You may keep this booklet.

4. The last page contains a word search and a Sudoku puzzle. These are not part of the competition; they are there for your enjoyment.

5. Put your name on the Scantron. Your last name must start in the left column of the last name section. You do not need to put blanks at the end of your name. Put your nine-digit student number in the Banner ID on the Scantron sheet. You **must** include the four zeros at the beginning of this number. If your name and number are not on the sheet your answers will not be graded.
   Example:



1) A parent is designing a swing set. The swing is to have 2 ft. of clearance with the ground, and will be attached using chains that are 8ft. long. If the legs of the set are to make 60^0 angles with the ground, what is the length of the legs?



A) $\frac{20}{\sqrt{3}}$ ft.

B) $\frac{10\sqrt{2}}{3}$ ft.

C) $\frac{8\sqrt{2}}{5}$ ft.

D) $8\sqrt{3}$ ft.

E) 13 ft.

2) A secret message is being sent using a code that uses two digit numbers to represent each letter. The sum of the two digit numbers representing each letter is put at the end of each coded word to ensure accuracy. Which of the following is a valid code?

A) 23581

B) 561213

C) 111220

D) 6814

E) 1112131349

3) Jane is playing the lottery; she buys 50 tickets at $2 per ticket. Each time Jane plays she uses 50% of any money that she wins to purchase more $2 lottery tickets and the rest of her winnings go in the bank. If every time Jane purchases lottery tickets she wins back 50% of her purchase; how much money has she lost after she has finished playing?

A) $34

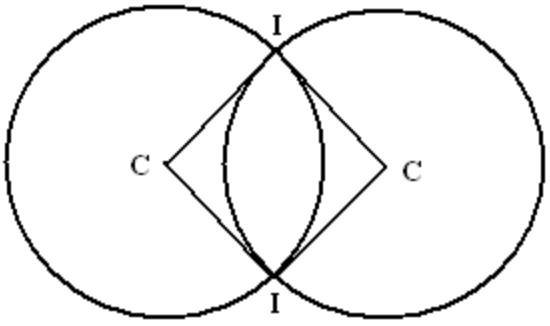
B) $50

C) $65

D) $66

E) $84

4) Two circles of radius r are placed so that they overlap, as shown in the figure. A figure is drawn connecting the centers C of the two circles to the points of intersection I. For this figure to be a square the distance between the centers of the two circles has to be:



A) 2

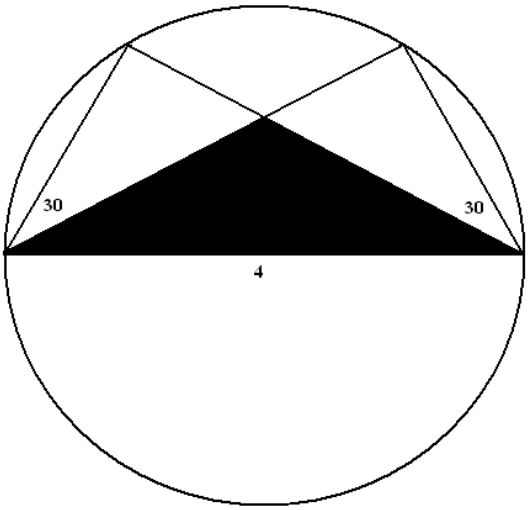
B) $\sqrt{r}$

C) $\sqrt{2r}$

D) $\sqrt{2}\,r$

E) $\sqrt{\pi r}$

5) Two congruent triangles are inscribed in a circle of diameter 4; the intersection of the two triangles is shaded. At the vertices the two triangles have in common the sides of the triangles make a 30^o angle with each other. Find the area of the shaded region.



A) $\frac{2}{\sqrt{3}}$

B) $\frac{3}{2}$

C) $\frac{1}{\sqrt{2}}$

D) $\frac{4}{\sqrt{3}}$

E) $\frac{\sqrt{3}}{2}$

6) If *f(n)*= $\frac{1}{3}n(n+1)(n+2)$, then $f(r) - f(r-1)$ equals

A) $r(r+1)$

B) $(r+1)(r+2)$

C) $\frac{1}{3}r(r+1)$

D) $\frac{1}{3}(r+1)(r+2)$

E) $\frac{1}{3}r(r+1)(r+2)$

7) The nonnegative integers are aligned in columns as shown. Which column contains 999?

A) A

B) B

C) C

D) D

E) E

A	B	C	D	E
2	1	0		
	3	4	5	6
10	9	8	7	
	11	12	13	14
18	17	16	15	
	19	20	…	

8) The product along any row, column, or diagonal of the 3×3 square is the same nonnegative integer. Find *x*.

A) 15

B) 25

C) 75

D) 125

E) 375

15		***x***
25	**5**	

9) The hare can run 100 feet in the time it takes the tortoise to run 60. How many feet must the hare run in order to catch the tortoise if the tortoise has a 100-foot head start?

A) The hare never catches the tortoise.

B) 250 ft.

C) 235 ft.

D) 195 ft.

E) 160 ft.

10) Each of the five letters in the cryptarithm correspond to one and only one of the digits 2, 4, 6, 8, and 9. Determine the digit to which E corresponds.

A) 2

B) 4

C) 6

D) 8

E) 9

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  ONE
+ ONE
  TWO
```

11) How many 2-digit whole numbers are equal to three times the product of their digits?

A) 0

B) 1

C) 2

D) 3

E) 4

12) If $ax^3 - bx^2 + cx - d = (x - 4)(x^2 + 3x + 2) - (x + 2)(x^2 - 3x + 2)$ for all real numbers x, then what is $a + b + c + d$?

A) 0

B) 2

C) 4

D) 6

E) 8

13) Consider the three numbers ar, ar^2, and ar^3. If their sum is 12 and the sum of their reciprocals is 3, then their product is

A) 3

B) 4

C) 8

D) 12

E) 36

14) Instead of studying for the exam two calculus students go to a concert and stay up all night. As a result they both sleep through the exam. Both students go to Dr. Goodguy and tell him that they got a flat tire and could not get back in time to take the exam. Dr. Goodguy, being a good guy, allows the students to take the exam. On the exam there is only one question, "Which tire?" What is the probability that the students pass the exam if they are in different rooms?

A) $\frac{1}{4}$

B) $\frac{1}{5}$

C) $\frac{1}{8}$

D) $\frac{1}{16}$

E) $\frac{1}{20}$

15) If you list the first 1000 integers consecutively

1234567…500501502503504…9989991000

how many 55's appear in this list if single digits are allowed to be in more than one pair?

A) 11

B) 13

C) 27

D) 31

E) 33

16) A billiards player stands at pocket P of a pool table and aims for the point Q on the opposite rail, which is at a distance x from the corner pocket R. On a regulation 4x8 table, how long should x be so that the ball hits three different rails and goes into pocket R?

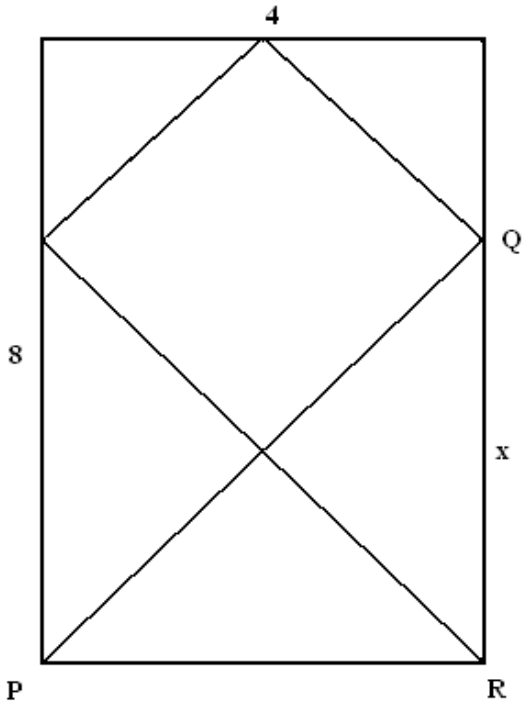
A) $\frac{16}{3}$ft.

B) $\frac{8}{3}$ ft.

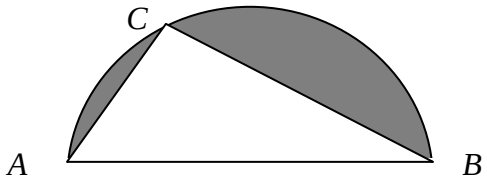
C) 4ft.

D) $3\sqrt{2}$ ft.

E) 6 ft.



17) The diameter *AB* of the semicircle has length 5 and the segment *BC* has length 4. Find the area of the shaded region.



A) $\frac{25\pi-96}{8}$

B) $\frac{25\pi-48}{8}$

C) $\frac{25\pi-48}{4}$

D) $\frac{25\pi-96}{4}$

E) $\frac{50\pi-48}{4}$

18) A container in the shape of a right circular cylinder of diameter three inches has one inch of water in it. All of this water is poured into another cylindrical container. This taller cylinder has a diameter of two inches. How deep is the water?

A) $\frac{2}{3}$ inches

B) $\frac{4}{3}$ inches

C) $\frac{3}{2}$ inches

D) 2 inches

E) $\frac{9}{4}$ inches

19) If 6!7! = *n*!, find *n*. (Note: *k*! = $1 \cdot 2 \cdot 3 \cdot \cdots \cdot k$. For example $3! = 1 \cdot 2 \cdot 3 = 6$)

A) 8

B) 9

C) 10

D) 11

E) 12

20) If Mr. Pfister leaves home at 7:30 and drives 40 mph, he arrives at work one minute late. Leaving at the same time but driving at 45 mph, he arrives one minute early. How far does Mr. Pfister drive to work?

A) 6 miles

B) 12 miles

C) 17 miles

D) 18 miles

E) 42.5 miles

21) Your vet has a sedative mixed with saline water, 100 ml combined, so that the concentration of the sedative in water is 0.5%. To give your dog a shot the solution needs to be at a concentration of 0.05%. How much water needs to be added to the 100 ml to get it to 0.05%?

A) 500 ml

B) 900 ml

C) 1000 ml

D) 1250 ml

E) 1700 ml

22) A circle intersects the center of a larger circle which has twice the radius. Shade in the overlap of the two circles. Subtracting the overlap from the area of the larger circle gives a multiple of the area of the smaller circle. This multiple is:

A) $\frac{1}{2}$

B) 1

C) 2

D) 3

E) π

23) The ones digit of $(((((7^1)^2)^3)^4)^{\cdots})^{2008}$ is:

A) 1

B) 3

C) 4

D) 7

E) 9

24) Randomly write down the digits 1,2,3, and 4. As you write the digits down, compute the partial sums by adding all digits written down so far (so there will be 4 partial sums). The probability that the partial sums are **never** a multiple of 3 is:

A) 0

B) $\frac{1}{3}$

C) $\frac{1}{4}$

D) $\frac{1}{2}$

E) $\frac{2}{3}$

25) You build a robot which builds a replica of itself in one week. It does this for 3 weeks and then it retires. Each new robot does the same. After 4 weeks how many robots are there including retired robots?

A) 10

B) 15

C) 16

D) 32

E) 60

26) You are on a game show. There are 4 doors, and behind one is a car. You choose 1 door and then the host opens one of the other three doors that does not have the car. The host asks if you would like to switch your choice to one of the other 2 remaining doors, which you do. Your probability of now winning the car is:

A) 0

B) $\frac{1}{4}$

C) $\frac{3}{8}$

D) $\frac{1}{2}$

E) $\frac{3}{4}$

27) A guy goes on vacation to a resort and has eight vouchers he can use for either golf in the morning or a massage in the afternoon. Five of the mornings he was there he slept in, seven of the afternoons he was there he went swimming, and on some of the days he did not use a voucher. What is the smallest number of days was he there?

A) 8

B) 9

C) 10

D) 11

E) 12

28) If we increase all the sides of a cube by 3 inches, the volume increases by 63 inches. What was the original side?

A) 1 in.

B) 2 in.

C) 3 in.

D) 4 in.

E) 5 in.

29) If $f(2)=1,\ \ f(1)=0$, and $f(n)=f(n-1)+f(n-2),$ for $n\geq 3$, what is $f(5)$?

A) 2

B) 3

C) 4

D) 5

E) 6

30) A small grocery store had 10 cartons of milk, 2 of which were sour. If you are going to buy the sixth carton of milk sold that day at random, what is the probability of selecting a carton of sour milk?

A) $\frac{1}{2}$

B) $\frac{1}{3}$

C) $\frac{1}{4}$

D) $\frac{1}{5}$

E) $\frac{1}{6}$