Division of Mathematics and Computer Science
Alfred University

Alfred, NY 14802

Instructions:

1. This competition will last seventy-five minutes – from 10:05 to 11:20.

2. The use of calculators is not permitted.

3. There are thirty questions. Mark your answers on the computer answer sheet. Use a #2 pencil only. You may use this question booklet for scratch work. You may keep this booklet.

4. The last page contains a word search. The word search is not part of the competition; it is there for your enjoyment.

5. Put your name on the Scantron. Your last name must start in the left column of the last name section. You do not need to put blanks at the end of your name. Put your nine-digit student number in the Banner ID on the Scantron sheet. You **must** include the four zeros at the beginning of this number. If your name and number are not on the sheet your answers will not be graded.

1) A Latin square of size 5 is a square grid containing the symbols A, B, C, D, E so that each letter appears exactly once in each column and exactly once in each row. Fill in the empty squares of this grid to make a latin square. Which symbol appears in column 2, row 4?

col 1	col 2	col 3	col 4	col 5	
A		C		E	row 1
C		E	A	B	row 2
E			C	D	row 3
					row 4
			B	C	row 5

col 1   col 2   col 3   col 4   col 5

A) A

B) B

C) C

D) D

E) E

2) Jim encodes a message by replacing each letter with a number:
                    23 36 23 40 26 35 25 29 26 33 37.
Which of the messages below is Jim's?
A) SOS send help.

B) SOS math help.

C) Bob send milk.

D) Bob send help

E) Owe that hurt

3) The ratio of the circumference of the outer circle to the sum of the circumferences of the eight inner circles is:
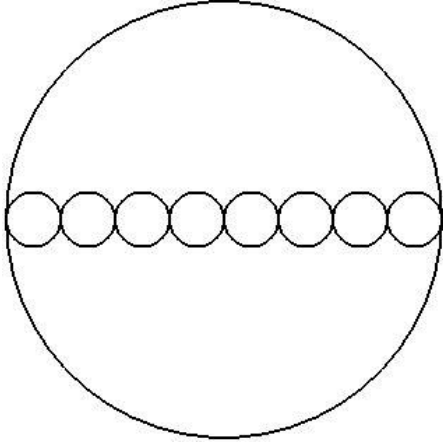
A) 1

B) 1.25

C) 1.5

D) 1.75

E) 2

4) In a standard deck of 52 playing cards what is the smallest number of cards you must draw to guarantee that you will have six of one suit? (Remember, a standard deck of cards has four suits with 13 cards in each suit.)

A) 20

B) 21

C) 23

D) 24

E) 25

5) Define operation $\otimes$ by $a \otimes b = \dfrac{a}{b} + \sqrt{a+b}$. Compute $(12 \otimes 4) \otimes 2$.

A) $2 + \sqrt{6}$

B) $7 + \sqrt{7}$

C) 13/2

D) 7

E) 4

6) How many 6's are there as digits in a listing of the numbers from 1 to 100?

A) 10

B) 19

C) 20

D) 25

E) 30

7) The volume of a rectangular box whose side, front, and bottom faces are 12 square inches, 8 square inches, and 6 square inches, respectively, is

A) 9 cubic inches

B) 24 cubic inches

C) 96 cubic inches

D) 104 cubic inches

E) 576 cubic inches

8) You and your friend have a total of $100 between you. Your friend borrowed ¼ of your share two days ago and borrowed $10 more yesterday. How much money did you have to start if today you both have the same amount?

A) $40

B) $50

C) $60

D) $70

E) $80

9) There are nine stones. Eight of the stones weigh the same and one of the stones weighs slightly more than the rest. If you use a balance scale, what is the minimum number of weighings that you need to find the heavier stone?

A) 1

B) 2

C) 3

D) 4

E) 5

10) Two high school soccer players, Mia and Abby, decide to jog around the new triangular race tracks at their school. The race tracks are similar. Mia can jog one lap around race track A in 2 minutes. If Abby jogs at the same rate, how long will it take her to finish one lap around race track B?

.25 mi

Track A

.35 mi
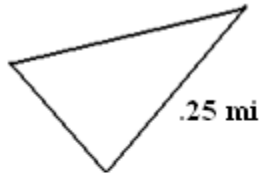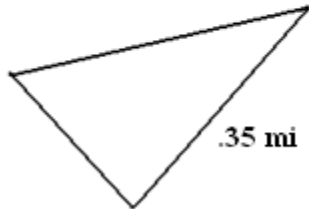
Track B

A) 3 minutes

B) 2 minutes

C) 10/7 minutes

D) 14/5 minutes

E) 3/2 minutes

11) If it were two hours later, it would be half as long until midnight as it would be if it were only an hour later. What time is it now?

A) 1 am

B) 4 am

C) 11 pm

D) 9 pm

E) 10 pm

12) There are 100 senators at a meeting. We are given the following two facts:
(i) Each senator is either crooked or honest.
(i) At least one of the senators is honest.
(ii) Given any two of the senators at least one of the two is crooked.
How many of the senators are honest?

A) 1

B) 10

C) 20

D) 50

E) Can't be determined.

13) Stack four isosceles, right triangles to form the region shown. If the legs of the smallest triangle measure $\sqrt{2}$ units, then the total area of the region is

A) $15\sqrt{2}$

B) 15

C) $\dfrac{15}{\sqrt{2}}$

D) 10

E) $\dfrac{15}{2}$
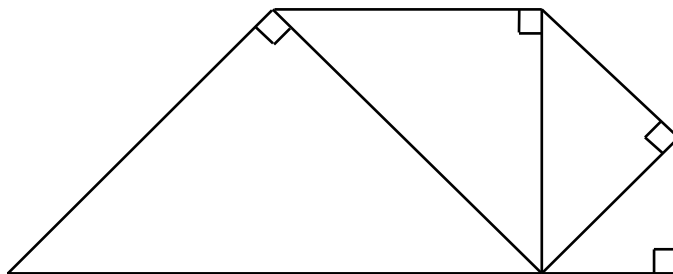
14) Alice is shooting free throws. She hits her first and misses her second. After that the probability she makes her next shot is equal to the proportion of hits so far. What is the probability she hits exactly half of her first six shots?

A) 1

B) 1/2

C) 1/3

D) 1/4

E) 1/5

15) If $x + y = 3$ and $x^3 + y^3 = 9$, then $x^2 + y^2 =$

A) 3

B) 5

C) 6

D) 7

E) 9

16) Putting a seven between the digits of a certain two-digit number is 540 less than putting a 7 in front of the number, but 36 more than putting a 7 at the end. The two-digit number is

A) 12

B) 13

C) 21

D) 23

E) 31

17) You have two cups: one holds exactly 5 ounces of water and the other holds exactly 11 ounces of water. There are no markings on the cups and you are not allowed to mark the cups in any way. You are given a huge bucket of water. Of the following, which can you measure with what you have been given?

A) 1 oz. and 6 oz.

B) 2 oz. and 3oz.

C) 12 oz.

D) A and C

E) all of the above.

18) A goat is chained at the top of a conical hill with a 20 ft chain. The hill is 80 ft tall and has a radius of 60 ft at its base. If the goat walks around the hill once in a circle at the full length of his chain, how far did he walk?

A)20 ft.

B) 24 ft.

C) 20π ft.

D) 24π ft.

E) 144π ft.

19) Batwoman walks 7 m north from her current position, jumps 8 m onto the roof of a building, walks 4 m east, jumps down 2 m onto the roof of an adjacent building, and walks another 2 m north and then 2 m west. How far is she from her original position?

A) 11 m

B) $\sqrt{141}$ m

C) $\sqrt[3]{141}$ m

D) $\sqrt{93}$ m

E) 20 m

20) A survey of 100 commuters show the following: 55 drove the freeway, 49 wore seatbelts, 42 listened to the radio, 20 listened to the radio and wore seat belts, 29 listened to the radio and drove the freeway, 31 drove the freeway and wore seat belts, and 12 did all three. How many did none of these?

A) 14

B) 18

C) 20

D) 22

E) 24

21) The average of 50 numbers is 26. Two of these numbers are 32 and 68. What will the average of the 48 remaining numbers be if we remove 32 and 68?

A) 20

B) 24

C) 25

D) 26

E) 30

22) A circular table is pushed into a corner in a rectangular room so that it touches both walls. The circular table touches a smaller rectangular table in the corner that is 9 units long and 2 units wide. What is the radius of the circular table?

A) 5 units

B) 12 units

C) 15 units

D) 17 units

E) 20 units
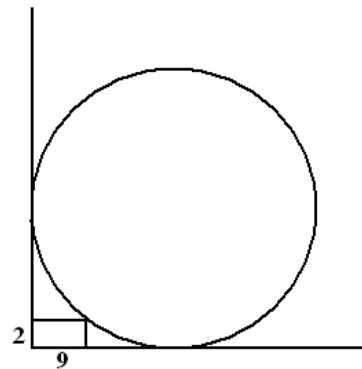
23) If $x + y = 0$ and $x \neq 0$, then $\dfrac{(x + 2y)^{2007}}{(2x + y)^{2007}} =$

A) 1

B) 1/2

C) 0

D) −1/2

E) −1

24) A pole is standing vertically in a lake in such a way that one-fifth of the pole is in the mud, two-thirds is in the water, one-eighth is above the water, and a piece of the top, measuring one foot and three inches, is broken off. What is the depth of the lake?

A) 50 feet

B) 75 feet

C) 100 feet

D) 125 feet

E) 150 feet

25) Two circles with radii 2 cm and 3 cm are externally tangent. A third larger circle is circumscribed about these as shown. The ratio of the area of the smallest circle to the area of the shaded region is
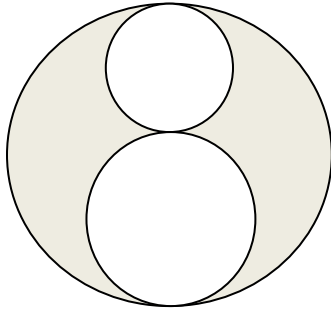
A) 1/9

B) 2/9

C) 1/3

D) 2/5

E) 2/9

26) Two squares are inscribed inside a circle as shown. The distance between the centers of the large 2×2 square and the smaller square is
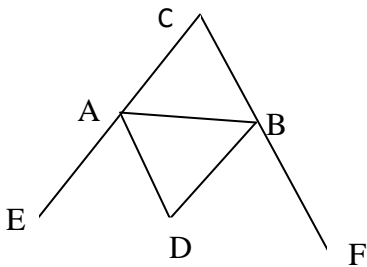
A) 22/20

B) 23/20

C) 24/20

D) 25/20

E) 26/20
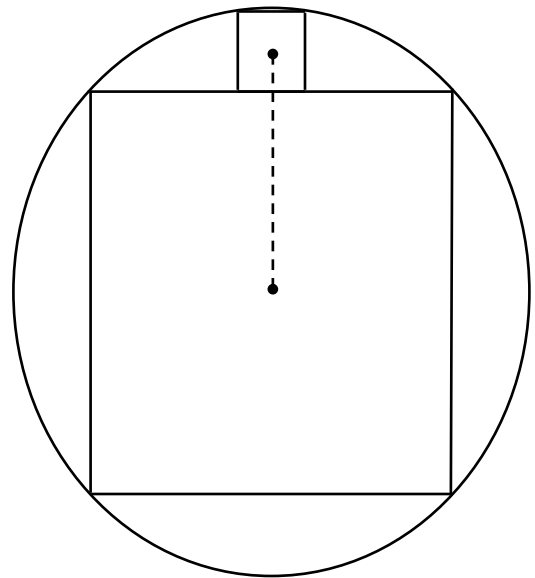
27) The units digit of $3^{2007} - 2^{2007}$ is

A)1

B) 3

C) 5

D) 7

E) 9

28) In the figure below angle ACB measures $40°$, angle EAD equals angle DAB, and angle FBD equals angle DBA. The measure of angle ADB is

A) 50°

B) 60°

C) 70°

D) 80°

E) 90°

29) Anna had a 10 gallon keg of wine and a jug. One day, she drew off a jugful of wine from the keg and replaced it with pure water. Later, when the wine and the water were thoroughly mixed, she drew off another jugful from the keg and replaced it with pure water. The keg then contained equal quantities of wine and water. What was the capacity of the jug?

A) $\frac{5}{2}$ gal.

B) $10 - 2\sqrt{5}$ gal.

C) 5 gal.

D) $10 - 5\sqrt{2}$ gal.

E) $\frac{15}{2}$ gal.

30) A certain rare disease occurs in 1 out of 1000 people. One hundred thousand people get tested for the disease and your test comes back positive, indicating you could have the disease. Your doctor tells you that the test is very reliable: 95% of the sick people will test positive and 95% of people without the disease will test negative. What is the probability you actually have the disease (rounded to the nearest whole percentage)?

A) 2%

B) 5%

C) 50%

D) 61%

E) 95%