AU

William Varick Nevins III High School Mathematics Competition
Fall 2005

Division of Mathematics and Computer Science
Alfred University
Alfred, NY 14802

Instructions:

1. This competition will last seventy-five minutes – from 10:05 to 11:20.

2. Put your five-digit student number in the correct place on the computer answer sheet.

3. The use of calculators is not permitted on this examination.

4. There are thirty questions. Mark your answers on the computer answer sheet. Use a #2 pencil only. You may use this question booklet for scratch work.

1. A four-foot diameter circular rug sells for $60. Assuming that the selling price is in direct proportion to the amount of material needed to make the rug, what will be the price of a six-foot diameter rug?

   A) $75

   B) $90

   C) $100

   D) $135

   E) $150

2. The isosceles triangle *ABC* illustrated below has height 30 cm and a base of 9 cm, the length of segment *CE* is 1/3 the length of segment *AC*, and *DE* is parallel to *BC*. What is the area of trapezoid *BCED*?

   A) 75 cm^2

   B) 105 cm^2

   C) 110 cm^2

   D) 120 cm^2

   E) 150 cm^2

   F) 190 cm^2

3. For the following cryptarithm puzzle, each letter represents a digit from 0-9, and different letters represent different digits; also the first digit in a number cannot be zero. The value of **N** is

$$\begin{array}{cccc} & A & H \\ + & H & A \\ \hline U & N & U \end{array}$$

A) 0

B) 2

C) 4

D) 6

E) 8

4. Ann and Sue bought identical boxes of stationery. Ann used hers to write 1-sheet letters and Sue used hers to write 3-sheet letters. Ann used all the envelopes and had 50 sheets of paper left, while Sue used all the sheets of paper and had 50 envelopes left. The number of sheets of paper in each box was:

A) 90

B) 105

C) 120

D) 125

E) 150

5. Four computers are to be networked together. Each of the computers may have multiple connections to any of the other computers. What is the minimal number of connections necessary so that if any one of the connections in the network is broken, all of the computers will still be connected to the network?

   A) 4

   B) 5

   C) 6

   D) 7

   E) 8

6. For Halloween you buy a bag of candy to give out. You wish to arrange the candy in equal groups, but if you arrange the candy in groups of 2, 3, 4, 5, or 6 you always have 1 piece left over. What is the smallest possible number of pieces of candy you have?

   A) 19

   B) 31

   C) 49

   D) 61

   E) 121

7. The diagonal of a square is equal to the diameter of a circle. The ratio of the area of the square to that of the circle is

   A) $\frac{1}{\pi}$

   B) $\frac{2}{\pi}$

   C) 1

   D) π

   E) $\frac{4}{\pi}$

8. An urn contains 20 marbles of which 10 are red, 8 are blue, and 2 are green. You are going to select marbles at random from this urn. What is the minimum number you can select without replacement and be **certain** that you have selected two of the same color?

   A) 2

   B) 3

   C) 4

   D) 11

   E) 20

9. If $x+\frac{1}{x}=y$, then what is $x^3+\frac{1}{x^3}$?

   A) y^3

   B) y^3-3y^2-6

   C) y^3-3y

   D) y^3-3y^2

   E) y^3+3y^2+3y+1

10. What is the length of a side of an equilateral triangle inscribed in a circle of radius 1?

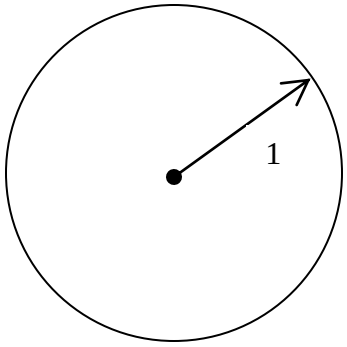
A) $\frac{\pi}{3}$

B) $\sqrt{2}$

C) $\frac{\pi}{2}$

D) $\sqrt{3}$

E) 2



11. Two jars each contain 100 marbles. Jar 1 contains all black marbles and Jar 2 contains all red. Select 25 marbles from Jar 2 and mix them into Jar 1. Now **randomly** select 25 marbles from the **well-mixed** Jar 1 and return them to Jar 2 (so both jars now have 100 marbles again but contain both red and black marbles). Which of the following is true:

A) The number of red marbles in Jar 1 is always greater than the number of black marbles in Jar 2.

B) The number of red marbles in Jar 1 is usually greater than the number of black marbles in Jar 2.

C) The number of red marbles in Jar 1 is always less than the number of black marbles in Jar 2.

D) The number of red marbles in Jar 1 is usually less than the number of black marbles in Jar 2.

E) The number of red marbles Jar 1 is always the same as the number of black marbles in Jar 2.

12. How many 2-digit numbers *n* have the property that 7 times the sum of the digits of *n* is *n*? (NOTE: Zero has one digit.)

A) 0

B) 1

C) 2

D) 3

E) 4

13. $\lceil x \rceil$ means round the number x to the next highest whole number. For example $\lceil 4.02 \rceil = 5$, and $\lceil 5 \rceil = 5$. For how many of the integers 1, 2, 3, …, 100 is $\left\lceil \frac{n}{3} \right\rceil$ divisible by 4?

A) 0

B) 8

C) 19

D) 24

E) 33

14. In circles with centers A and D, AB = DC = 1 and $\overleftrightarrow{BC}$ is a tangent line. The shaded regions have equal areas. Find BC.

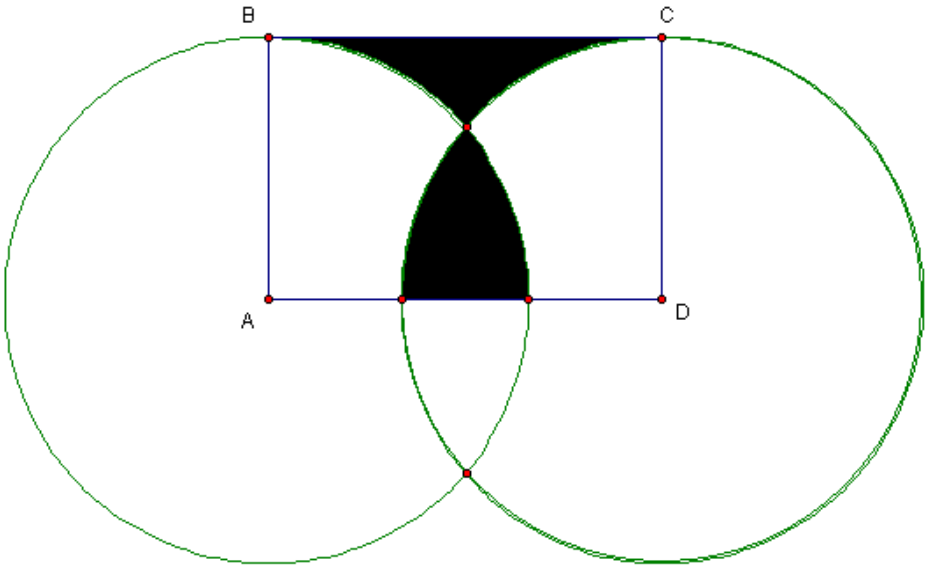
A) $\frac{\pi}{8}$

B) $\frac{4}{\pi}$

C) $\frac{\pi}{2}$

D) π

E) 2π



15. A teenage girl wrote her own age after her father's age. From this new 4 place number she subtracted the absolute value of the difference in their ages to get 4,289. The sum of their ages was

A) 48

B) 52

C) 56

D) 59

E) 64

16. If a pump and a half pumps a quart and a quarter in a second and a third, then how many quarts do two pumps pump per minute?

    A) 30

    B) 45

    C) 60

    D) 75

    E) 105

17. Ma ‘n’ Pa ‘n’ brother ‘n’ me,
    The sum of our ages is eighty-three.
    Six times Pa’s is seven times Ma’s,
    ‘N’ Ma’s is three times me.

    How old is brother? (All ages are integers.)

    A) 4

    B) 5

    C) 6

    D) 7

    E) 8

18. How many 3 digit numbers x have the following property?

    If you subtract 7 from x the difference is divisible by 7, if you subtract 8 from x the difference is divisible by 8, and if you subtract 9 from x the difference is divisible by 9.

    A) 0

    B) 1

    C) 2

    D) 3

    E) more than 3

19. If the line whose equation is $y = 4x + p$ intersects the parabola whose equation is $y = x^2 + x$ in exactly one point, then p is equal to

    A) $-\frac{9}{4}$

    B) $-\frac{4}{9}$

    C) 0

    D) 1

    E) $\frac{9}{4}$

20. The sum of the individual digits appearing in the list 1, 2, 3, ..., 99 is

    A) 445

    B) 450

    C) 495

    D) 855

    E) 900

21. Two sides of a triangle have lengths 4 and 7 and meet at an angle of 45°. The area of the triangle is

A) $2\sqrt{7}$

B) 7.5

C) $7\sqrt{2}$

D) 14

E) $14\sqrt{2}$

22. The shaded region is bounded by two equilateral triangles, each of side length 3. Find the area of the shaded region.

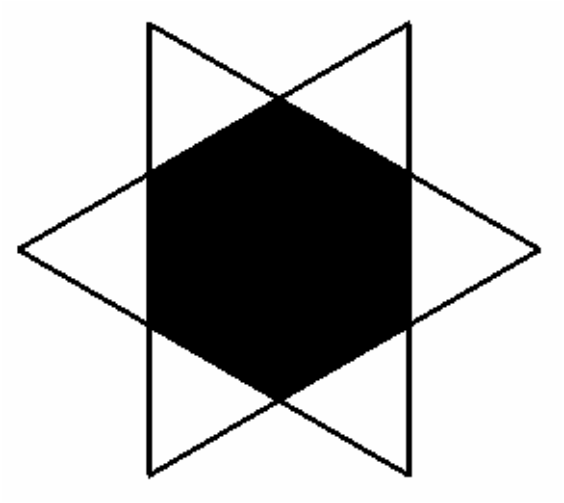
A) $\frac{3\sqrt{3}}{2}$

B) $\frac{5\sqrt{3}}{2}$

C) $5\sqrt{2}$

D) 8

E) $6\sqrt{2}$



23. There are 100 bowls lined up in a row and 100 students lined up with a bag of marbles. The first student places a marble in every bowl, the second student places a marble in every other bowl (starting with the second one), the third student in every third bowl starting with the third bowl, and so on. What is the largest number of marbles a bowl will contain?

A) 6

B) 8

C) 10

D) 11

E) 12

24. A farmer is mowing a square field of side length **S**, by mowing around the field from the outside in, creating a border of mowed grass. If the width of the mowed grass is given by **x**, what fraction of the side length will **x** be when the farmer has mowed half of the field?

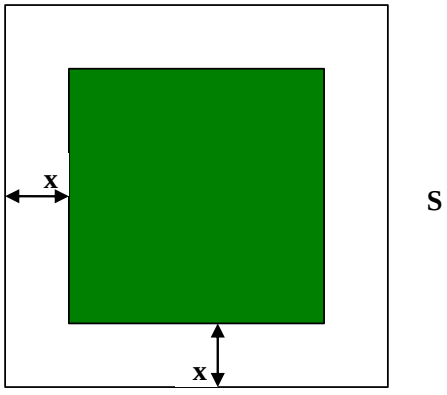
A) $\frac{2-\sqrt{2}}{4}$

B) $\frac{\sqrt{2}}{4}$

C) $\frac{1}{2}$

D) $\frac{2+\sqrt{2}}{4}$

E) 1



25. *A* and *B* travel around a circular track at constant but not necessarily equal speeds in opposite directions, starting from diametrically opposite points. If they start at the same time, meet first after *B* has traveled 100 yards, and meet the second time 60 yards before *A* completes one lap, then the circumference of the track in yards is:

A) 400

B) 440

C) 480

D) 560

E) 880

26. A set of numbers is said to be minimally self-contained if the number of elements in the set is an element of the set and no proper subset has this property. Which of the following sets are minimally self-contained?

$$S_1 = \{1, 2\}, \quad S_2 = \{2, 4\}, \quad S_3 = \{1, 2, 4\}, \quad S_4 = \{3, 4, 5\}$$

A) none of them

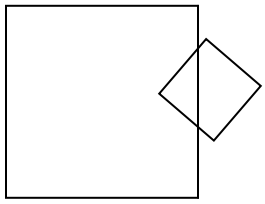
B) S_2 only

C) S_1 and S_2 only

D) S_2 and S_4 only

E) all of them

27. The squares shown have side lengths 1 and 3. The difference between the non-overlapping areas is



A) 1

B) 2

C) 4

D) 8

E) 9

28. You have 10 poker chips in your pocket: 1 $1 chip, 2 $2 chips, 3 $3 chips and 4 $4 chips. You reach into your pocket and remove 3 chips at random and without replacement. What is the probability that the sum of the values of the three chips is a multiple of 5?

A) $\frac{1}{240}$

B) $\frac{5}{72}$

C) $\frac{1}{5}$

D) $\frac{5}{24}$

E) $\frac{5}{8}$

29. If *b* is two more than *a* and three less than *c*, then what is $a^2b - a^2c + ac^2 - ab^2 + b^2c - bc^2$?

A) -30

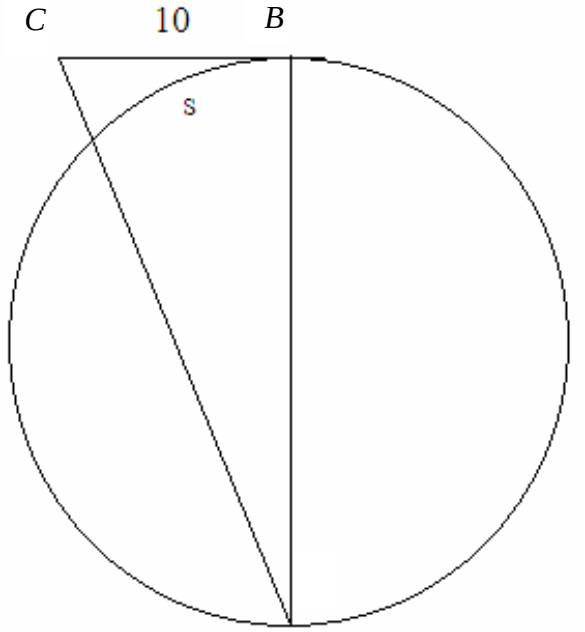
B) -15

C) -1

D) 5

E) 6

30. Arc s is $\frac{1}{6}$ the circumference of the circle, *AB* is a diameter of the circle and *BC* is tangent to the circle. What is the circumference?



A) $10\sqrt{3}\pi$

B) $\frac{20}{\sqrt{3}}\pi$

C) $\frac{15\sqrt{2}}{2}\pi$

D) 30

E) $\frac{10\sqrt{2}}{5} + \pi$