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Instructions:

1. This competition will last seventy-five minutes – from 10:05 to 11:20.

2. Put your five-digit student number in the correct place on the computer answer sheet.

3. The use of calculators is not permitted on this examination.

4. There are thirty questions. Mark your answers on the computer answer sheet. Use a #2 pencil only. You may use this question booklet for scratch work.

1. Let x denote how much larger $\dfrac{4}{3}$ is than $\dfrac{3}{4}$. Let y denote the ratio of $\dfrac{3}{4}$ to $\dfrac{4}{3}$. The larger of x and y is

   A) $\dfrac{9}{16}$

   B) $\dfrac{7}{12}$

   C) $\dfrac{3}{4}$

   D) $\dfrac{7}{9}$

   E) $\dfrac{4}{3}$

2. In a seven-legged race a team consists of people standing side by side who have their adjacent legs tied together which count as one leg. How many contestants are needed for two teams?

   A) 8

   B) 11

   C) 12

   D) 13

   E) 16

3. A two-player game begins with one rod of integer length. The players take turns breaking a rod into two rods of unequal integer lengths until all pieces have length 1 or 2, so no further breaks are possible and the game ends. The player who made the last move wins if most pieces have length 1 and loses if most pieces have length 2; the game is tied if there are equally many pieces of length 1 and length 2.

    Suppose the game begins with a rod of length 5. Choose the true statement:

    A) The game will always end in a tie regardless of what moves are made.

    B) Whoever goes first automatically wins regardless of what moves are made.

    C) Whoever goes second automatically wins regardless of what moves are made.

    D) Whoever goes first can win, but only if the other player makes a mistake.

    E) Whoever goes second can win, but only if the other player makes a mistake.

4. How many new images can be created from the image below by performing any combination of the operations
    i) Flip the image over the vertical axis
    ii) Flip the image over the horizontal axis
    iii) Rotate the image counterclockwise 90°?
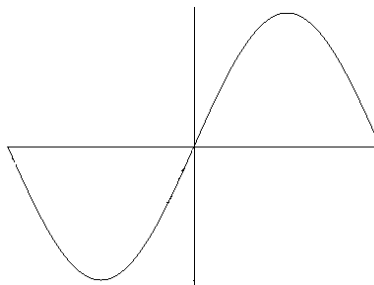
    A) 1

    B) 2

    C) 3

    D) 4

    E) 8

5. How many of the numbers $\sqrt{7}, \sqrt{10}, \sqrt{13}, \sqrt{16}, \cdots, \sqrt{148}$ are integers?

A) 0

B) 6

C) 10

D) 15

E.) 41

6. Each of a group of 50 rabbits is white or brown and is blue or brown-eyed. If 14 are blue-eyed white rabbits, and 31 are brown rabbits, and 18 rabbits have brown eyes, then the number of brown rabbits with brown eyes is:

A) 5

B) 7

C) 9

D) 11

E) 13

7. The number of real solutions of the equation $|x-1|+|x-2|=7$ is

A) 0

B) 1

C) 2

D) 3

E) infinite

8. A basketball player scored 23 points in a game on 2-point and 3-point shots only (no free throws). In how many different ways can this occur?

    A) 1

    B) 2

    C) 3

    D) 4

    E) 23

9. The game show "Most Extreme Elimination Challenge" has a contest with four walls, and four doors in each wall. In each wall there are three doors made from wood, and one made from paper. A contestant wins by guessing which of the doors are made of paper. If at any one of the walls they encounter a wooden door, then their turn is over and they are eliminated from the contest. If each of the doors is equally likely to be chosen, and there are 256 contestants per game, then how many paper doors can the MXC staff expect to replace in each game?

    A)   4

    B) 50

    C) 64

    D) 85

    E) 125

10. A closed box contains the 3 letters W, O, and W. You draw 3 letters, one at a time, replacing each letter after you have recorded it. If you are able to spell WOW with the three letters you drew, you win a prize! Wow! What is the probability of winning a prize?

    1. $\dfrac{4}{27}$

    2. $\dfrac{3}{8}$

    3. $\dfrac{4}{9}$

    4. $\dfrac{16}{27}$

    5. $\dfrac{2}{3}$

11. A 2x3 rectangle has vertices at (0, 0), (2, 0), (0, 3), and (2, 3). It rotates $90°$ clockwise about the point (2, 0). It then rotates $90°$ clockwise about the point (5, 0), then $90°$ clockwise about the point (7, 0), and finally, $90°$ clockwise about the point (10, 0). (The side originally on the x-axis is now back on the x-axis.) Find the length of the curve traced out by the point whose initial position is (1, 1).

    A) $\dfrac{\pi}{2}(\sqrt{2}+\sqrt{5})$

    B) $2\sqrt{2}\,\pi$

    C) $\dfrac{7}{2}\pi$

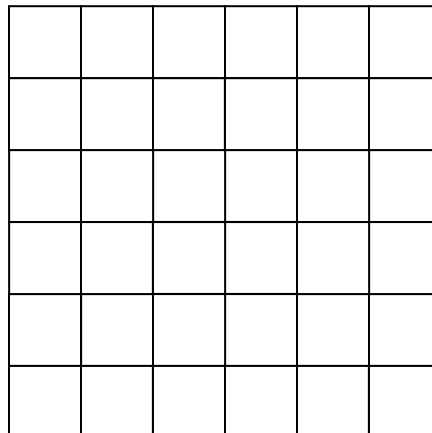   D) $\pi(\sqrt{2}+\sqrt{5})$

    E) $6+\dfrac{7}{2}\pi$

12. You drive to visit your grandparents and average 40 mph; on the way home you average 60 mph. What was your average speed for the trip?

    A) 45 mph

    B) 46 mph

    C) 48 mph

    D) 50 mph

    E) 52 mph

13. How many squares are there of any size in the figure below?

    A) 36

    B) 55

    C) 86

    D) 90

    E) 91

14. A sphere of radius r is sliced by 2 distinct planes, each containing the center of the sphere. This causes the sphere to fall apart into 4 distinct wedges. Calculate the combined surface area of all 4 wedges. (The surface area of a sphere of radius r is $4\pi r^2$.)
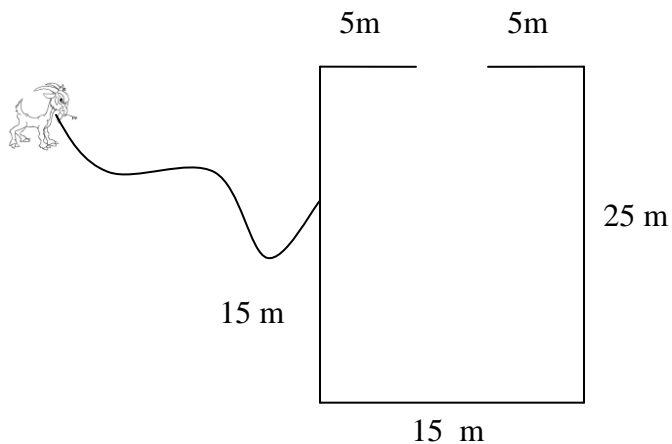
   A) $6\pi r^2$

   B) $8\pi r^2$

   C) $10\pi r^2$

   D) $12\pi r^2$

   E) $16\pi r^2$

15. A goat is tied to the side wall of a barn by a chain of length 20m. The dimensions of the barn are shown in the diagram below. The barn door is open as indicated by the gap in the wall. The total area over which the goat can roam is:

   5m   5m

   25 m

   15 m

   15 m

   A) 200π m^2

   B) 225π m^2

   C) 231.25π m^2

   D) 237.5π m^2

   E) 243.75π m^2

16. The number of positive integers less than 1000 divisible by neither 5 nor 7 is:

    A) 630

    B) 658

    C) 684

    D) 686

    E) 687

17. Suppose Amy is driving at a steady rate of 60 mph when Sarah passes her at 80 mph. Sarah's car breaks down ten miles after she passes Amy. How long after Sarah's car breaks down will Amy catch up to Sarah?

    A) 1 minute

    B) 2.5 minutes

    C) 5 minutes

    D) 7.5 minutes

    E) 10 minutes

18. If $x + y = 3$ and $x^2 + y^2 = 5$, then $x^3 + y^3 =$

    A) 7

    B) 9

    C) 11

    D) 13

    E) 19

19. AB is the diameter of the circle, with length 4. OP is a perpendicular bisector of AB, and AC is tangent to the circle. Find the area of triangle ABC.
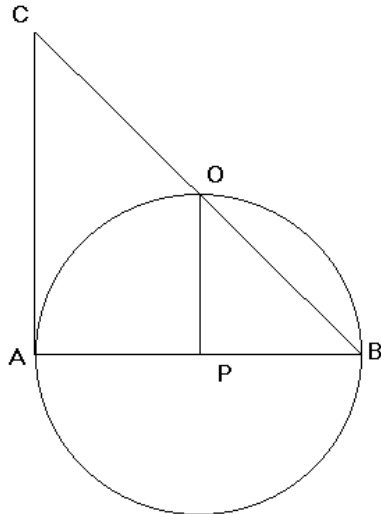
   A) 1

   B) 2

   C) 4

   D) 8

   E) 13

20. An equilateral triangle with a height of 1 is sliced by a line parallel to one side, dividing it into a trapezoid and a triangle. The area of the new small triangle is ½ the area of the original equilateral triangle. What is the height of the new small triangle?

A) $\dfrac{2-\sqrt{2}}{2}$

B) $\dfrac{1}{2}$

C) $2-\sqrt{2}$

D) $\dfrac{\sqrt{2}}{2}$

E) $\dfrac{2+\sqrt{2}}{2}$

21. If seven kids can eat 14 pizzas in three hours, how many pizzas can six kids eat in four and a half hours?

    A) 14

    B) 16

    C) 17

    D) 18

    E) 20

22. If the diameter of the circle is 4, then the area of the inscribed hexagon is


    A) $3\sqrt{3}$

    B) $6\sqrt{2}$

    C) $6\sqrt{3}$

    D) 12

    E) 4π
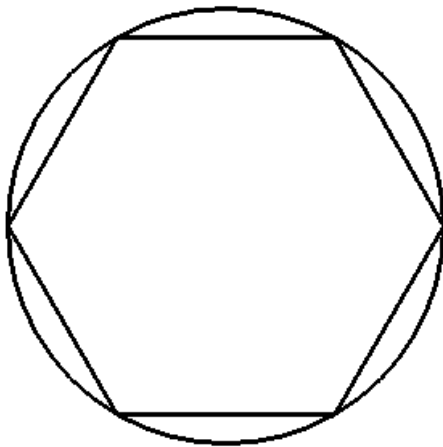
23. Bob is now half Betty's age. Six years ago, he was one-third her age. How old is Betty now?

    A) 21

    B) 22

    C) 24

    D) 26

    E) 28

24. 7^{2004} divided by 5 gives a remainder of

    A) 0

    B) 1

    C) 2

    D) 3

    E) 4

25. A set of four cards each has a letter on one side and a number on the other side. The sides we can see of these cards show E, 4, H, and 7. We suspect that all four cards obey the rule that if the letter side is a vowel, then the number on the other side is even. The minimal subset of the four cards that we <u>must</u> turn over in order to determine whether all four cards do, in fact, obey this rule is:

    A) E

    B) E and 4

    C) E and 7

    D) E, 4 and 7

    E) all four cards

26. If one writes the integers from 1 through 1000, how many digits are written?
    (10 has two digits, 100 has three.)

    A) 1000

    B) 2789

    C) 2792

    D) 2893

    E) 3000

27. The first Koch snowflake is an equilateral triangle. The second snowflake is obtained from the first by removing the middle third of each side of the triangle, and replacing this with two sides of the same length as the removed part. Continue this process as in the diagrams. How many corners will there be in the fifth snowflake?
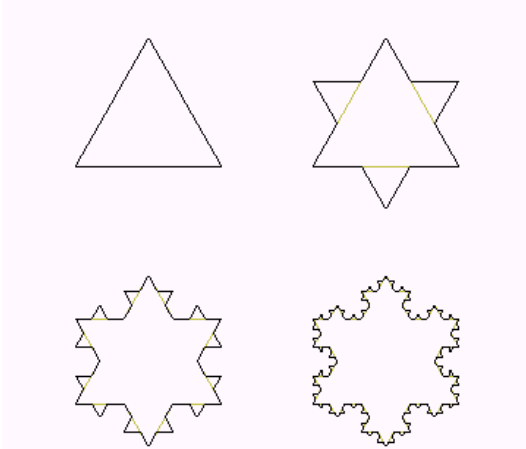

A)   192

B)   768

C)   3072

D) 12336

E) 49344

28. A car radiator is filled to its 5.1 liter capacity with a solution of 75 percent antifreeze and 25 percent water. The antifreeze manufacturer recommends a concentration of 50 percent antifreeze for summer driving. How much of the car's coolant should be drained and replaced with pure water to reduce the antifreeze concentration to 50 percent?

A) 1.25 liters

B) 1.275 liters

C) 1.5 liters

D) 1.7 liters

E) 2.55 liters

29. In the figure to the right AB is a
    diameter of length 5, chord AC has
    length 4, and CD perpendicular to AB.
    The length of AD is
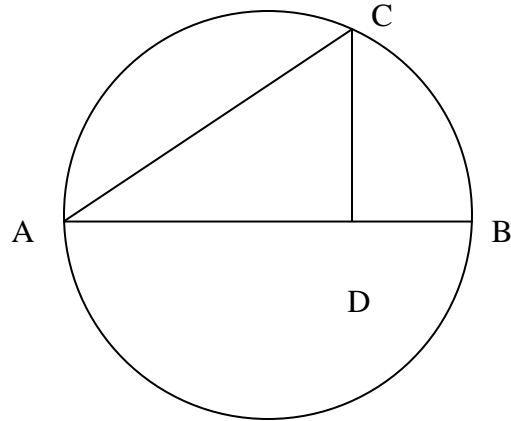
    A) 3

    B) $\dfrac{16}{5}$

    C) $\dfrac{7}{2}$

    D) $\dfrac{18}{5}$

    E) $\dfrac{19}{5}$

30. Each day a man's wife meets him at the railroad station and drives him home. One day he
    arrives at the station an hour early and begins to walk home along the road his wife always
    takes. She meets him en route and takes him the rest of the way home. Had he waited at the
    station, she would have picked him up exactly on time. As it turned out, he reached his home
    twenty minutes early. How long did he walk?

    A) 30 minutes

    B) 40 minutes

     C) 45 minutes

    D) 50 minutes

    E) 55 minutes