William Varick Nevins III High School Mathematics Competition
Fall 2003

Division of Mathematics and Computer Science
Alfred University
Alfred, NY 14802

Instructions:

1. This competition will last seventy-five minutes – from 10:05 to 11:20.

2. Put your five-digit student number in the correct place on the computer answer sheet.

3. The use of calculators is not permitted on this examination.

4. There are thirty questions. Mark your answers on the computer answer sheet. Use a #2 pencil only. You may use this question booklet for scratch work.

1. An item is marked down by 10% and then a week later the new price is marked down another 10%. This final price is what percentage of the original?

    A) 79%

    B) 80%

    C) 81%

    D) 85%

    E) 90%

5. A tube with circular cross section expands so that the radius of the cross section increases by 20%, by what percentage does the volume of the tube increase?

    A) π%

    B) 10%

    C) 20%

    D) 40%

    E) 44%

3. Starting with a list of the integers 1 through 100, cross out all the multiples of three. Next, cross out all of the remaining numbers that are multiples of two. How many additional numbers are crossed out in this second round?

    A) 10

    B) 22

    C) 29

    D) 34

    E) 37

4. Amy, Julie, and Maria each compete on a different athletic team and take a different science course. Amy does not know how to swim. No one on the volleyball team is taking chemistry. Julie is using her ticket to tonight's home basketball game as a bookmark in her physics book. Maria is planning to work on her chemistry homework on the bus on the way to tonight's away competition. Choose the true statement from the list below:

   A) Maria is on the swim team and Amy is taking biology.

   B) Amy is on the volleyball team and Maria is taking chemistry.

   C) Julie is on the basketball team and Amy is taking chemistry.

   D) Maria is on the basketball team and Julie is taking physics.

   E) Amy is on the swim team and Julie is taking biology.

5. What is the minimum number of colors required so that in the figure below no two adjacent regions are the same color?

   A) 2

   B) 3

   C) 4

   D) 5

   E) 6
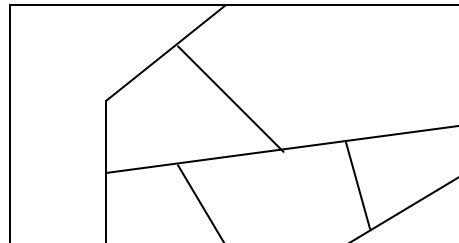
6. A square is inscribed in a circle. The ratio of the area of the square to that of the circle is

    A) $\dfrac{1}{2\pi}$

    B) $\dfrac{1}{\pi}$

    C) $\dfrac{\sqrt{2}}{\pi}$

    D) $\dfrac{2}{\pi}$

    E) $\dfrac{4}{\pi}$

7. Two circles with radii 2 and 3 cm are externally tangent. A third larger circle is circumscribed about these as shown. Find the ratio of the area of the smallest circle to the area of the shaded region.
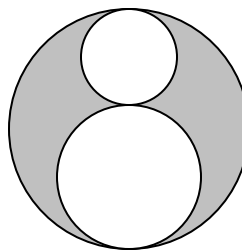
    A) 1/9

    B) 2/9

    C) 1/3

    D) 2/5

    E) 4/9

8. A dog has a rectangular fenced in area, 32 feet by 20 feet, in which it buried a bone but has forgotten where the bone is. If the dog can dig at a rate of 2 square feet per minute and it only has 2 and one-half hours to dig before its owner comes home, what is the probability, to the nearest percent, that the dog finds its bone?

    A) 19%

    B) 27%

    C) 38%

    D) 47%

    E) 58%

8. If $xy = 2$, $xz = 10$, and $xyz = 100$, what is yz?

    A)   5

    B)   10

    C)   20

    D)   25

    E) 500

10. Second graders have to memorize the addition table for the numbers 1 through 10. Assuming they know that addition is commutative, how many addition facts must they memorize?

    A)   45

    B)   50

    C)   55

    D)   60

    E) 100

11. A train one mile long travels at 30 mph. It enters a tunnel one mile long at 4 PM. At what time does the end of the train emerge from the tunnel?

    A) 4:02 PM

    B) 4:04 PM

    C) 4:15 PM

    D) 4:30 PM

    E) 5:00 PM

12. The area of a right triangle with one leg of length 30 and altitude to the hypotenuse of length 24 is
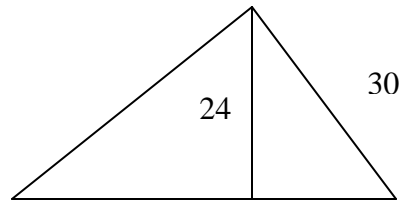
    A) 300

    B) 600

    C) 900

    D) 1200

    E) 1500

13. Mark and Jesse are fishing in a canoe half of a mile from shore. The canoe springs a leak and water starts pouring in at rate of 10 gallons per minute. The canoe will sink if it takes on more than 30 gallons of water. If Mark starts bailing water at a rate of 6 gallons per minute, how fast must Jesse paddle the canoe so that they reach the shore before they sink?

    A) 6 miles per hour

    B) 0.5 miles per hour

    C) 4 miles per hour

    D) 7.5 miles per hour

    E) They will sink regardless of how fast Jesse paddles

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14. Find the area of the entire rectangle.

A) 40

B) 45

C) 59

D) 99

E) 104

12	20
27	

15. The points with coordinates (x_1, y_1) and (x_2, y_2) are not horizontally aligned. The x-intercept of the line containing these two points is

A) $\dfrac{x_1 \cdot y_2 - y_1 \cdot x_2}{y_2 - y_1}$

B) $\dfrac{y_1 \cdot y_2 - x_1 \cdot x_2}{y_2 - y_1}$

C) $\dfrac{y_1 \cdot x_2 - x_1 \cdot y_2}{x_2 - x_1}$

D) $\dfrac{y_2 - y_1}{x_2 - x_1}$

E) $\dfrac{x_2 - x_1}{y_2 - y_1}$

16. In the diagram showing two concentric circles, the smaller circle has a radius one-half of the larger circle's radius. A circular sector cut from the large circle is be split into two portions: a sector of the small circle, A, and another region, B. Find the ratio of the areas of the two portions to each other, $\dfrac{A}{B}$.
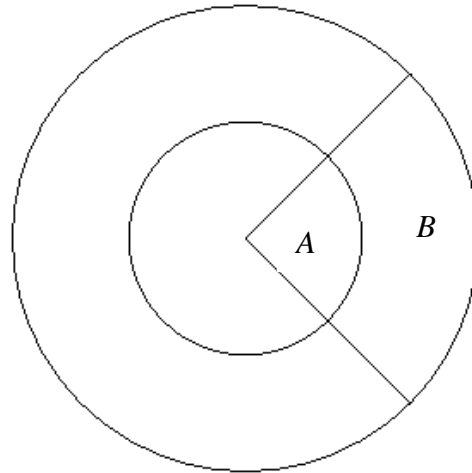
A) $\dfrac{1}{6}$

B) $\dfrac{1}{5}$

C) $\dfrac{1}{4}$

D) $\dfrac{1}{3}$

E) $\dfrac{1}{2}$

17. Let x be the year of your birth. Perform the following operations in order: square x and then subtract 25. Divide that quantity by the year that is five years prior to your birthday. Now subtract 5 years from this result, subtract x from this quantity, and then multiply this by 2003. The result is

A) Zero

B) Your age

C) Your score on this years Nevin's exam plus 10

D) The year you were born

E) This year

18. Ed's grandmother drove her motorcycle to his house at 60 mph. She then returned home on the same road at 90 mph. If her total time on the road was 2 hours, how far apart do they live?

    A) 50 miles

    B) 60 miles

    C) 72 miles

    D) 84 miles

    E) 90 miles

19. If five positive integers are chosen at random, what is the probability that the product of these numbers is even?

    A) 30%

    B) 50%

    C) 75%

    D) 97%

    E) 99%

20. A pipe of radius $\dfrac{2}{\pi}$ inches and length 5 feet is to be covered with strips of tape 1/4 inch wide by winding the strips of tape around the pipe, edge to edge. How much tape will be needed?

    A) 650 inches

    B) 960 inches

    C) 1012 inches

    D) 340π inches

    E) 355π inches

21. A goat is tethered to the side of a barn by a chain 30 feet long. If the chain is attached at a point 10 feet from the corner of the barn and the barn is 60 feet long and 40 feet wide, find the total area in which the goat can graze.

A) 500π square feet

B) 550π square feet

C) 600π square feet

D) 650π square feet

E) 700π square feet

40'

10'

60'
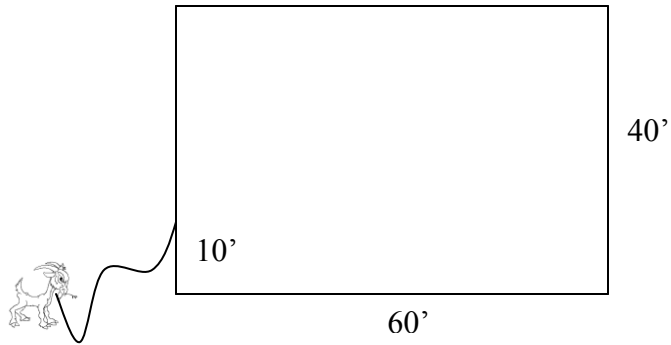
22. In a square of area 1, lines *EF* and *GH* are parallel to the same diagonal and divide the square into three regions of equal area. Find the length of *EF*.

A) $\dfrac{1}{3}$

B) $\dfrac{1}{2}$

C) $\dfrac{\sqrt{3}}{3}$

D) $\dfrac{\sqrt{6}}{3}$

E) $\dfrac{2\sqrt{3}}{3}$

23. The diagram below represents a valve of radius $r = 1$ inch, currently opened as far as possible with its handle parallel to the wall. To close the valve, we turn the handle of length $h = \sqrt{3}$ inches to the right until the end of the handle hits the wall, rotating the valve clockwise. If the distance between the wall and the valve is $d = \sqrt{2} - 1$ inches, through how many degrees can the valve be rotated?

    A) 20

    B) 30

    C) 45

    D) 60

    E) 75
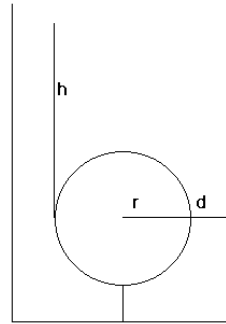
24. Compute the sum $3 + 7 + 11 + 15 + \cdots + 399$

    Hint: $1 + 2 + 3 + \cdots + n = \dfrac{1}{2}n(n+1)$

    A) 17,100

    B) 17,900

    C) 18,900

    D) 19,100

    E) 20,100

25. Two equilateral triangles, each of side length 12, overlap to form a regular six-pointed star. What is the area of the intersection of the two triangles?

    A) 8

    B) $8\sqrt{3}$

    C) 24

    D) $24\sqrt{3}$

    E) $48\sqrt{3}$

26. A circle of radius *r* has a wedge removed from it. The long side of the wedge is on a diameter of the circle, and the interior angle of the wedge is 45 degrees. If the circle has radius r, what is the area of the wedge?
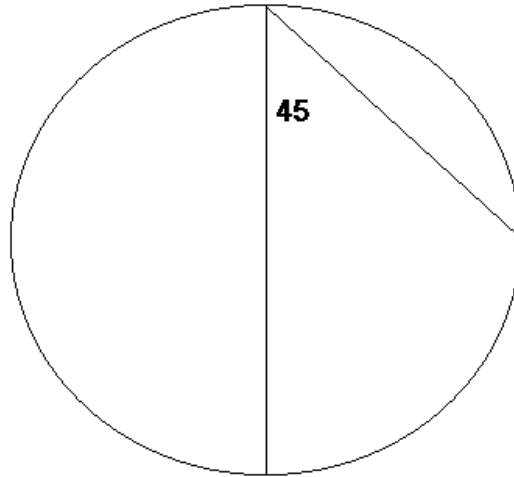
A) πr^2

B) $2r^2$

C) $\dfrac{r^2}{4}(\pi - 2)$

D) $\dfrac{r^2}{4}(\pi + 2)$

E) $\pi r^2 \tan(45^\circ) + \dfrac{1}{3}\pi r\cos(45^\circ)$

27. Determine the remainder when 76^{2003} is divided by 35.

A) 0

B) 1

C) 3

D) 6

E) 33

28. Let $\{\,x\,\}$ denote the non-negative distance between real number x and its nearest integer neighbor. (For example, $\{\,5.7\,\} = 0.3$.) Compute the sum of $\{\,n/5\,\}$ from $n = 1$ to 20.

    A) 1.2

    B) 3.6

    C) 4

    D) 4.8

    E) 24

29. Al and Ben play a game involving a series of coin tosses. Each gambler picks a different series of three possible outcomes, and the winner is the one whose chosen series of outcomes first occurs in the string of coin tosses. Al announces that his series is HHH. If Ben makes an optimal choice for his series, what is his probability of winning?

    A) 0.5000

    B) 0.6250

    C) 0.7500

    D) 0.8750

    E) 0.9375

30. An isosceles triangle with one side of length 4 and two sides of length x is inscribed in a circle of radius 4. The value of x is

A) $\sqrt{32+8\sqrt{3}}$

B) $\sqrt{32+16\sqrt{3}}$

C) $\sqrt{16+32\sqrt{3}}$

D) $\sqrt{8\pi+32\sqrt{2}}$

E) $\sqrt{32+8\pi\sqrt{2}}$