AU

William Varick Nevins III High School Mathematics Competition
Fall 2002

Division of Mathematics and Computer Science
Alfred University
Alfred, NY 14802

Instructions:

1. This competition will last seventy-five minutes – from 10:05 to 11:20.

2. Put your five-digit student number in the correct place on the computer answer sheet.

3. The use of calculators is not permitted on this examination.

4. There are thirty questions. Mark your answers on the computer answer sheet. Use a #2 pencil only. You may use this question booklet for scratch work.

1. A cord of wood is a pile 8 ft. long, 4 ft. wide and 4 ft. high. How many cords are in a pile 20 ft. long, 12 ft. wide and 6 ft. high?

   A) $6\frac{1}{4}$

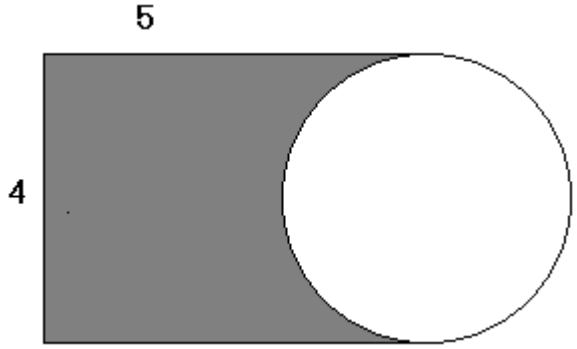
   B) 9

   C) $11\frac{1}{4}$

   D) $12\frac{3}{16}$

   E) 16

2. What is the area of the shaded region bounded by the rectangle and the circle to the nearest 0.01?



   A) 6.43

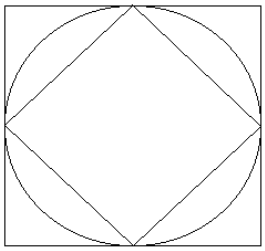
   B) 9.86

   C) 12.82

   D) 13.72

   E) 16.86

3. The ratio of the area of the smaller square to the larger square is:



A) $\frac{1}{4}$

B) $\frac{1}{\pi}$

C) $\frac{1}{3}$

D) $\frac{1}{2}$

E) $\frac{2}{\pi}$

4. The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$. If a sphere is inscribed in a cube with edge length 1, the volume of the sphere is

A) $\frac{\pi}{8}$

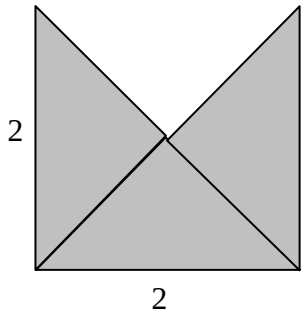
B) $\frac{\pi}{6}$

C) $\frac{\pi}{2}$

D) $\frac{2}{3}\pi$

E) $\frac{4}{3}\pi$

5. Find the shaded area.



A) 2

B) 2.5

C) 3

D) 3.5

E) 4

6. Three dice are tossed. What is the probability that the numbers shown will all be different?

A) $\frac{1}{36}$

B) $\frac{1}{9}$

C) $\frac{1}{2}$

D) $\frac{5}{9}$

E) $\frac{5}{6}$

7. A See-and-Say toy has 12 pictures of animals arranged in a circle and a pointer that can be rotated to any picture. When the lever is pulled, the toy makes the noise of the animal in the selected picture and automatically advances the pointer by a fixed number of pictures. Pulling the lever 12 times reaches every animal exactly once. Which of the following could be the number of pictures the pointer rotates each time the lever is pulled?

   A) 2

   B) 3

   C) 4

   D) 5

   E) 6

8. Suppose a waffle cone is held upright ready to fill with ice cream If the cone is 6 cm across at the top and 5 cm across at a depth of 2 cm from the top, how deep is the cone?

   A) 6 cm

   B) 8 cm

   C) 9 cm

   D) 10 cm

   E) 12 cm

9. The equation, $y = a(x-b)^2 + c$, where *a*, *b*, and *c* are non-zero real numbers, has two distinct real roots if:

   A) $a \cdot c < 0$

   B) *a*, *b*, and *c* are all positive

   C) *a* and *c* are both negative

   D) $b^2 - 4 \cdot a \cdot c > 0$

   E) $a \cdot b > 0$

10. A square has the property that its area is numerically equal to its perimeter. The diagonal of this square has length

A) $\sqrt{2}$

B) 2

C) $2\sqrt{2}$

D) 4

E) $4\sqrt{2}$

11. The equation $\frac{1}{a}+\frac{1}{b}=\frac{1}{a+b}$ is true for

A) all positive real numbers a and b

B) no positive real numbers a and b

C) exactly one pair of positive reals and b

D) exactly two pairs of positive reals a and b

E) infinitely many but not all pairs of positive reals and b

12. A pair of positive numbers has the property that their sum is twice their difference, while their product is twice their sum. The larger of these two numbers is

A) $\frac{1}{3}$

B) 2

C) $\frac{8}{3}$

D) $\frac{16}{3}$

E) 8

13. An altitude h of a triangle is increased by a length m. How much must be taken from the corresponding base b so that the area of the new triangle is that of the original triangle?

A) $\frac{bm}{h+m}$

B) $\frac{bh}{2(h+m)}$

C) $\frac{b(2m+h)}{m+h}$

D) $\frac{b(m+h)}{2m+h}$

E) m

14. A vending machine which accepts only quarters, dimes and nickels has its "*EXACT CHANGE ONLY . . . EXAC*" message flashing. An item in the machine costs \$0.65. What is the largest amount of money (in these three coins) one could have and still not be able to buy the item without overpaying?

A) \$0.60

B) \$0.70

C) \$0.75

D) \$0.80

E) There is no largest amount.

15. Tom, Dick and Harry mow lawns in the summer to earn extra money. They each have a lawnmower, and one Saturday they decide to mow a 5900 sq. ft. lawn together, using all three mowers. Tom mows 70 sq. ft./min., Dick 50 and Harry 40. Dick and Harry start mowing the lawn at the same time, but Tom has trouble starting his mower and is delayed for 30 minutes. All three boys stop mowing at the same time – when the lawn is finished. How long does Tom mow?

A) 10 minutes

B) 20 minutes

C) 30 minutes

D) 40 minutes

E) 50 minutes

16. The equation $\left(x^2-1\right)^{2002}\left(x^2-2\right)^{2002}\left(x^3+8\right)^{2002}=0$ has how many distinct integer solutions?

A) 1

B) 2

C) 3

D) 2^{2002}

E) 3^{2002}

17. A lattice point refers to a point (x, y) in the coordinate plane in which both coordinates, x and y, are integers. Consider the triangle having vertices at $(0, 0)$, $(10, 0)$ and $(0, 5)$. The number of lattice points strictly inside this triangle is

A) 16

B) 18

C) 20

D) 22

E) 24

18. A circle in the *xy*-plane passes through the three points (0, 3), (1,1) and (0, 0). The radius of this circle is

A) $\frac{\sqrt{5}}{2}$

B) $\frac{3}{2}$

C) $\frac{\sqrt{10}}{2}$

D) $\sqrt{5}$

E) $2\sqrt{2}$

19. How many 0's are at the end of the product of the first fifty positive integers?

A) 2

B) 5

C) 8

D) 10

E) 12

20. By weight, whole milk is 3% fat and 7% protein and carbohydrates; the other 90% is water. One gram of fat has nine calories, proteins and carbohydrates have four calories per gram, while water has zero calories per gram. What is the percentage of fat in milk by calories?

A) 3%

B) 23%

C) 49%

D) 65%

E) 80%

21. A car travels at 55 mph for half of the distance it covers, and 65 mph for the other half. What is the car's average velocity, rounded to the nearest tenth?

A) 57.0 mph

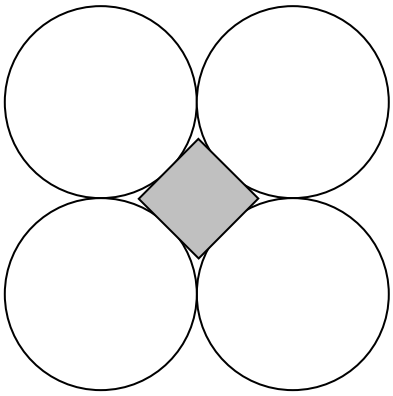
B) 57.2 mph

C) 59.6 mph

D) 60.0 mph

E) 61.2 mph

22. Four circles of radius 1 with centers at the corners of a square are tangent in pairs. Find the area of the inscribed square that fits between the circles.



A) $(2\sqrt{2}-2)^2$

B) $2\sqrt{2}-2$

C) 1

D) $2\sqrt{2}$

E) $(2\sqrt{2}-1)^2$

23. In the league below each team played each other once. The total points a team scored in all 3 games are listed in the FOR column, and the total points a team's opponents scored in all 3 games are listed in the AGAINST column:

Team	Win	Loss	Tie	For	Against
Alfred	**2**	**0**	**1**	**4**	**1**
Hornell	**2**	**1**	**0**	**3**	**1**
Wellsville	**0**	**1**	**2**	**0**	**2**
Olean	**0**	**2**	**1**	**1**	**4**

What was the score of the Alfred/Hornell game?

A) 0-0

B) 1-0

C) 2-1

D) 3-1

E) 2-0

24. A dog is chained to a tree 3 feet from the corner of a square pond (as shown in picture). The dog's chain is 6 feet long and the sides of the pond are 10 feet. What is the area of the pond that the dog can swim in?

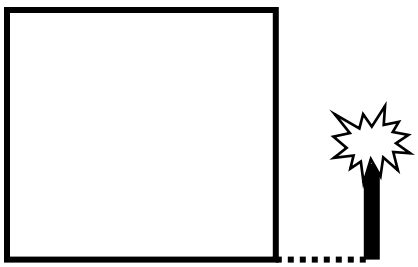
A) $3\pi - \frac{9\sqrt{3}}{2}$

B) $6\pi - \frac{9\sqrt{3}}{2}$

C) $9\pi - \frac{9\sqrt{3}}{2}$

D) $12\pi - \frac{9\sqrt{3}}{2}$

E) $36\pi - \frac{9\sqrt{3}}{2}$



25. The number 30 has eight positive factors; namely, 1, 2, 3, 5, 6, 10, 15, and 30. What is the sum of all positive two-digit integers with exactly twelve positive factors?

A) 0

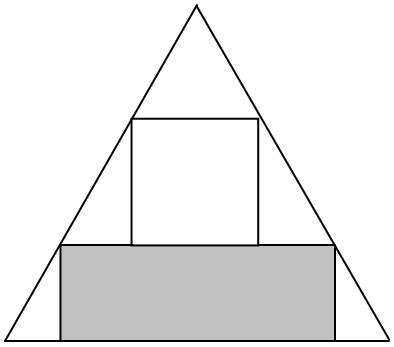
B) 132

C) 222

D) 306

E) 402

26. If the big triangle is equilateral, the unshaded rectangle is a square of side length 4, and the shaded rectangle has height 3, then the area of the shaded rectangle is



A) $12+4\sqrt{3}$

B) $12\sqrt{3}$

C) 24

D) $12+8\sqrt{3}$

E) 36

27. The last digit of $\left(\left(\left(7^7\right)^7\right)^7 \cdots\right)^7$, where 7 appears as an exponent 1000 times, is

A) 1

B) 3

C) 5

D) 7

E) 9

28. If $9^{x+2} = 240 + 9^x$ then x =

A) $\frac{1}{10}$

B) $\frac{1}{5}$

C) $\frac{3}{10}$

D) $\frac{2}{5}$

E) $\frac{1}{2}$

29. You have 7 cookie jars containing 1, 2, 3, …, 7 cookies respectively (i.e. jar 3 has 3 cookies in it). The cookie monster may choose any collection of jars and remove the **same** number of cookies from each jar in that collection. What is the fewest number of moves in which the cookie monster can empty all the jars?

    A) 1

    B) 2

    C) 3

    D) 4

    E) 5

30. A 4" x 4" x 4" cube is painted and then cut into sixty-four 1" x 1" x 1" unit cubes. A unit cube is then randomly selected and rolled. What is the probability that the top face of the rolled cube is painted?

    A) $\frac{1}{64}$

    B) $\frac{1}{32}$

    C) $\frac{1}{6}$

    D) $\frac{1}{4}$

    E) $\frac{1}{2}$