



**Division of Mathematics and Computer Science  
Alfred University  
Alfred, NY 14802**

**Instructions:**

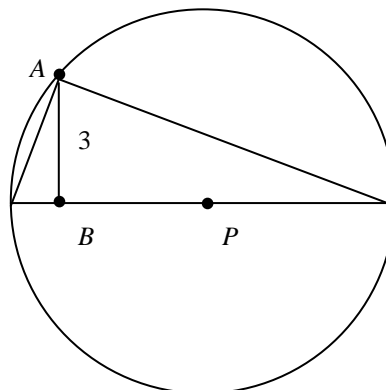
1. This competition will last seventy-five minutes – from 10:05 to 11:20.
2. Put your five-digit student number in the correct place on the computer answer sheet.
3. The use of calculators is not permitted on this examination.
4. There are thirty questions. Mark your answers on the computer answer sheet. Use a #2 pencil only. You may use this question booklet for scratch work.

1. A student chooses 16 whole numbers. For each of the numbers  $n$ , she then computes  $n(n+1)$ . What percentage of the 16 computed numbers  $n(n+1)$  will be even?
  - A) 0%
  - B) 25%
  - C) 50%
  - D) 75%
  - E) 100%
  
2. For New York driver licenses issued before September of 1992, the first three of the last five digits encoded the gender and birthday of the driver. For a driver born on day  $d$  of month  $m$ , these three digits are  $63m + 2d$  for a male and  $63m + 2d + 1$  for a female. Which of the following three digit numbers could come up in this way?
  - A) 189
  - B) 198
  - C) 891
  - D) 918
  - E) 981
  
3. The interest charged on a loan is  $p$  dollars per \$1000 for each of the first three months and  $q$  dollars per \$1000 for each month thereafter. How much interest will be charged during the first five months on a loan of \$15,000?
  - A)  $3p + 2q$
  - B)  $30p + 20q$
  - C)  $45p + 30q$
  - D)  $30p + 45q$
  - E)  $2p + 3q$

4. How many real solutions  $(x, y)$  are there to the system of equations

$$x^2 - y^2 = 0, \quad x - y = 1?$$

- A) 0  
 B) 1  
 C) 2  
 D) 3  
 E) infinitely many
5. Six people compete in a contest that lasts 12 days. Every 3 days one of the contestants is eliminated until the last day, when there are three contestants left. At the start of the contest the contestants are given 216 cups of rice. If the contestants eat 3 meals of rice per day, and they all get the same amount of rice at every meal, how many cups of rice should each contestant eat per day?
- A) 1  
 B) 2  
 C) 3  
 D) 4  
 E) 5
6. In the diagram, the circle has center  $P$  and radius 5. Segment  $AB$  is perpendicular to segment  $BP$ . Find the length of segment  $BP$ .



- A) 1  
 B) 2  
 C) 3  
 D)  $\sqrt{10}$   
 E) 4

7. In the game of Buzz, players count out loud together, taking turns each saying one number. However, if the number is a multiple of seven or has a seven in it, a player must say, "Buzz" instead of the number. So the game starts out: 1, 2, 3, 4, 5, 6, Buzz, 8, 9, 10, 11, 12, 13, Buzz, 15, 16, Buzz, 18, 19, 20, Buzz, 22, ... How many times should "Buzz" be said while counting from 1 to 100 in this way?

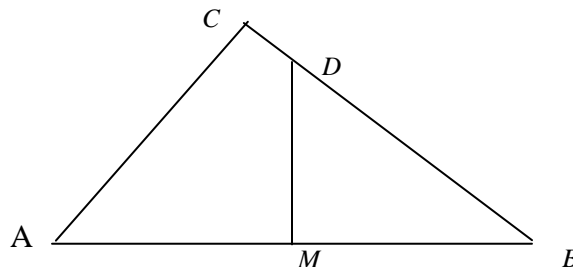
- A) 14 times
- B) 22 times
- C) 24 times
- D) 30 times
- E) 34 times

8. What is the units digit of  $3^{2001}$  ?

- A) 1
- B) 3
- C) 5
- D) 7
- E) 9

9. Angle  $ACB$  is a right angle. Segment  $DM$  is a perpendicular bisector of  $AB$ .  $AC$  has length 12;  $AB$  has length 20. The length of  $DM$  is

- A) 7
- B)  $7\frac{1}{2}$
- C) 8
- D)  $8\frac{1}{2}$
- E) 9



10. Let  $x_1 = 1$ ,  $x_2 = 2$ , and  $x_n = (x_{n-2} + x_{n-1}) / 2$ , for  $n = 3, 4, \dots$

Then  $x_{10} - x_9 =$

A) 1

B)  $\frac{1}{2}$

C)  $\frac{1}{10}$

D)  $\frac{1}{90}$

E)  $\frac{1}{256}$

11. The diameter of a circle is divided into two different lengths  $x$  and  $y$ , with a perpendicular line segment,  $h$ , drawn from the diameter to the circle as in the diagram. The length of  $h$  is

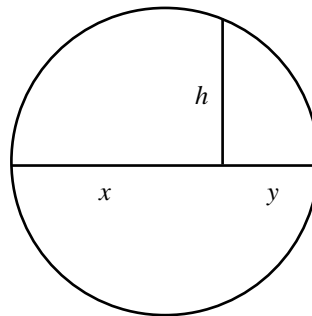
A)  $\frac{x+y}{2}$

B)  $\sqrt{x+y}$

C)  $\frac{xy}{2}$

D)  $\sqrt{xy}$

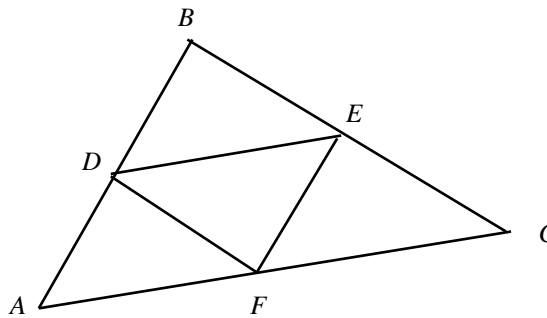
E) impossible to determine



12. An investment gains 80% in the first year, but the following year it loses 40%. The net result for the two years is
- A) no gain or loss
  - B) 40% gain
  - C) 4% loss
  - D) 8% gain
  - E) 8% loss

13. Given **any** triangle,  $ABC$ , we can form another triangle,  $DEF$ , by connecting the midpoints of the sides of triangle  $ABC$  as in the diagram. If we denote the measure of an angle by  $m\angle EDF$ , then  $m\angle EDF =$

- A)  $m\angle ABC$
- B)  $m\angle BCA$
- C)  $m\angle BAC$
- D)  $m\angle DEF$
- E)  $m\angle DFE$



14.  $|||-2|-2|-2|| =$
- A) -8
  - B) 0
  - C) 2
  - D) 4
  - E) 8

15. The line with negative slope  $m$  and positive  $y$ -intercept  $(0, b)$  determines a right triangle in the first quadrant as shown. The area of this triangle is

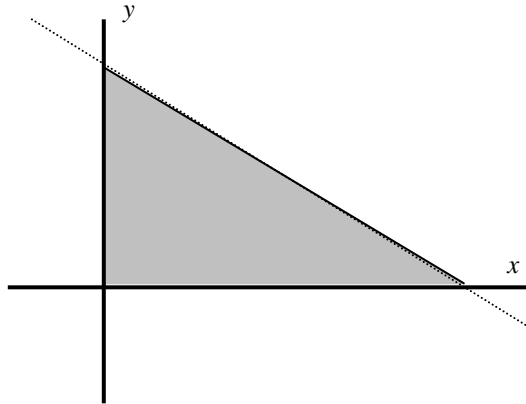
A)  $-\frac{b^2}{2m}$

B)  $\frac{b^2}{2m}$

C)  $\frac{mb}{2}$

D)  $\frac{2m}{b^2}$

E)  $-\frac{2m}{b^2}$



16. When a 20-foot cable is suspended between two points on a ceiling, the cable sags a bit in the center. When the midpoint of the cable is pulled down as far as possible, it is 3 feet below the ceiling. The distance between the points is between

A) 15 and 16 feet

B) 16 and 17 feet

C) 17 and 18 feet

D) 18 and 19 feet

E) 19 and 20 feet

17. If a tree 20 inches in diameter and 30 feet high increases its diameter two inches in one year and its height does not change, then what percentage growth in the volume of the tree does this represent?

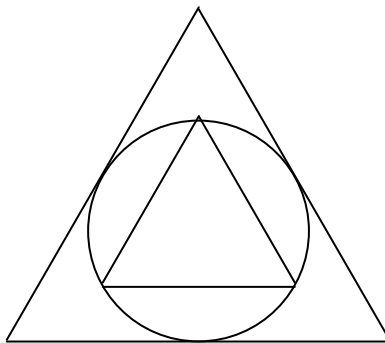
- A) 10%
- B) 16%
- C) 20%
- D) 21%
- E) 24%

18. If nine people are in a room and every person shakes hands exactly once with each of the other people, how many handshakes will occur?

- A) 8
- B) 15
- C) 36
- D) 72
- E) 81

19. If both triangles are equilateral and the sides of the largest triangle have length 2, what is the area of the smaller triangle?

- A)  $\sqrt{3}/4$
- B)  $\sqrt{2}/3$
- C) 1
- D)  $\sqrt{5}/2$
- E) 2.5







20. How many positive numbers less than 413 have an integer square root?
- A) 5
  - B) 10
  - C) 20
  - D) 30
  - E) 60
21. Each of six friends has a juicy piece of gossip. They are eager to share their gossip with the others. Whenever two of them get on the telephone, the conversation lasts an hour, during which each tells the other everything he or she knows so far. What is the minimum number of hours before each of them knows all six pieces of gossip?
- A) 2
  - B) 3
  - C) 4
  - D) 5
  - E) 6

22. A man is walking toward a streetlight hanging 20 feet above the ground. He pauses for a moment and notices that his shadow, which is behind him, is exactly as long as he is tall. He continues, walking under the lamp, until he notices that his shadow (now ahead of him) is again as long as he is tall. He has walked 27 feet in this time. How tall is he?

A)  $7\frac{1}{4}$  feet

B)  $5\frac{1}{2}$  feet

C)  $6\frac{1}{2}$  feet

D)  $5\frac{9}{20}$  feet

E) impossible to determine

23. In the figure  $ABCD$  is a parallelogram and  $|EF| = 9$  Find  $|FC|$ .

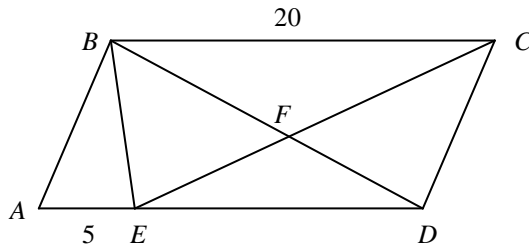
A) 12

B) 13

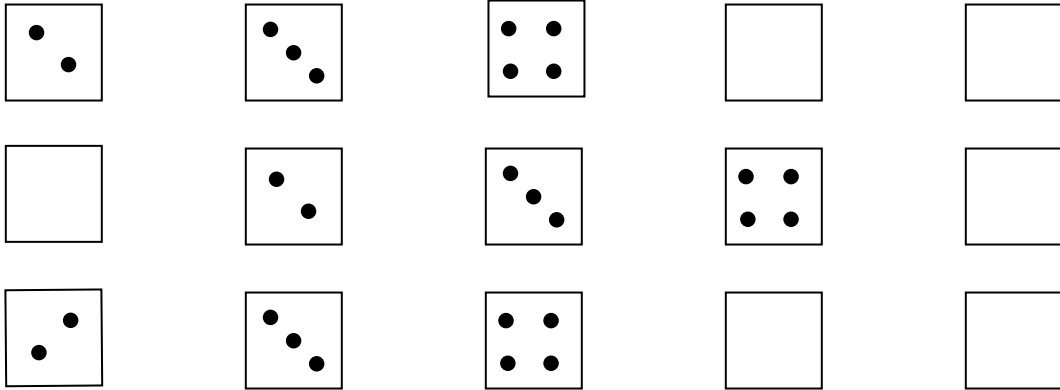
C) 14

D) 15

E) 16



24. Consider a game in which a player rolls five standard 6-sided dice, sets some of the dice aside in specially designated compartments, and re-rolls the rest. After her first roll, Amy has decided to keep a 2, a 3 and a 4. She could place them in the compartments in many ways that look different from the top, including these three:



In all, how many different ways could she place them?

- A) 60
  - B) 120
  - C) 240
  - D) 360
  - E) 480
25. The largest prime divisor of  $2^{14} + 2^8 + 1$  is
- A) 2
  - B) 3
  - C) 43
  - D) 61
  - E) 129

26. A man walking his dog along a straight country road spots a friend one mile away. His dog sprints ahead to meet her. When the dog reaches her, it turns back to the man. When it reaches him, it sprints back to the woman. The process continues until the two people meet. If each person walks at 2 mph and the dog runs at 8 mph, how far does the dog run before the two people meet?
- A) 1 mile
  - B) 1.5 miles
  - C) 2 miles
  - D)  $2\frac{1}{3}$  miles
  - E) 3 miles
27. The sum of two numbers is 30 while the sum of their squares is 522. Their product is
- A) 189
  - B) 200
  - C) 225
  - D) 15660
  - E) impossible to determine
28. A unit fraction is a proper fraction with a numerator of 1. How many ways can one express  $\frac{4}{5}$  as the sum of three different unit fractions?
- A) 0
  - B) 1
  - C) 2
  - D) 3
  - E) more than 3

29. A square of side length one is “rolled” along a straight line in the sense that it pivots  $90^\circ$  about each of its vertices in turn. Find the length of the path traced out by one of the vertices through one complete revolution.

A)  $\frac{(1+\sqrt{2})\pi}{2}$

B) 4

C)  $\frac{(2+\sqrt{2})\pi}{2}$

D)  $2\pi$

E)  $\frac{(3+\sqrt{2})\pi}{2}$

30. A ball was floating in a lake when the lake froze. The ball was removed (without breaking the ice), leaving a depression 24 cm across at the top and 8 cm deep. What was the radius of the ball (in cm)?

A) 12

B) 13

C) 15

D) 17

E) impossible to determine