



**William Varick Nevins III High School Mathematics Competition
Fall 2000**

**Division of Mathematics and Computer Science
Alfred University
Alfred, NY 14802**

Instructions:

1. This examination will last seventy-five minutes – from 10:05 to 11:20.
2. Put your five-digit student number in the correct place on the computer answer sheet.
3. The use of calculators is not permitted on this examination.
4. This examination contains thirty questions. Mark your answers on the computer answer sheet. Use a #2 pencil only. You may use this question sheet for scratch work.



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1. Assume the earth is a perfect sphere with radius 21 million feet. Suppose a string runs around the equator at a uniform height of 5 feet above the earth. Approximately how much longer is this string than one that lies on the surface around the equator?
 - A) 5 feet
 - B) 31 feet
 - C) 1 million feet
 - D) 6 million feet
 - E) 15 million feet

2. An automated tollbooth accepts only nickels, dimes, and quarters. How many different ways is it possible to pay a fifty cent toll with exact change?
 - A) 7
 - B) 8
 - C) 9
 - D) 10
 - E) 11

3. Let n be the number of real values of p for which the roots of $x^2 - px + p = 0$ are equal. Then n equals
 - A) 0
 - B) 1
 - C) 2
 - D) 4
 - E) an infinitely large number

4. A digital 12-hour clock is defective; the reading for hours is always correct, but the reading for minutes is always identical to that for hours. What is the minimum number of minutes between possible correct readings on the clock?
- A) 49
B) 50
C) 60
D) 61
E) 480
5. At Juan's Pizzeria a medium pizza has a diameter of 10 inches. It has been said that the amount of pizza in one-third of the medium pizza is the same as the amount of pizza in one-quarter of the large pizza. If this is true, what is the diameter of the large pizza?
- A) $\frac{10\sqrt{3}}{3}$ in.
B) $\frac{10\pi}{3}$ in.
C) 11 in.
D) $\frac{20\sqrt{3}}{3}$ in.
E) 12 in.
6. Assume that you invest an amount of money and in the first year your investment gains 50%. Unfortunately in the second year you lose 50%. If the amount of money after 2 years is \$900, how much money did you invest originally?
- A) \$600
B) \$675
C) \$900
D) \$1125
E) \$1200



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7. Suppose initially the number 3 is in Box 1 and the number 4 is in Box 2. Two operations can be performed:
- the two numbers can be added with the sum replacing the number in Box 1, or
 - the two numbers can be multiplied with the product replacing the number in Box 1.

If each of these operations is performed twice, which order produces the largest number in Box 1?

- A) multiply, multiply, add, add
 - B) add, multiply, add, multiply
 - C) multiply, add, multiply, add
 - D) add, add, multiply, multiply
 - E) multiply, add, add, multiply
8. If x and y are integers such that $x - y = 4$, how many possible values does the expression $\frac{2^{2x}}{4^y}$ have?
- A) None
 - B) One
 - C) Two
 - D) Three
 - E) Infinitely many



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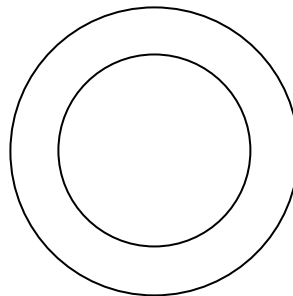
9. A telephone bill y (in dollars) is determined by $y = mx + b$, where x is the number of calls during a month and b is a fixed monthly charge. One customer made 8 calls and the total bill was \$12.00. Another customer made 15 calls and received a bill of \$15.50. What should be the bill for a customer making 20 calls in a month?
- A) \$17.00
B) \$18.00
C) \$19.00
D) \$20.00
E) \$21.00
10. Ten pennies are arranged in two stacks, one with 6 pennies and the other with 4. We remove one coin from each stack and use them to create a new stack.. Then we repeat this operation until we reach an arrangement which never changes (the order of the stacks is unimportant). What is that arrangement?
- A) 4, 4, 2
B) 5, 5
C) 5, 3, 2
D) 2, 2, 2, 2, 2
E) 4, 3, 2, 1

11. Glass A contains 4 ounces of soda; glass B contains 4 ounces of water. We transfer 1 ounce of soda from glass A to glass B, then stir the mixture in glass B thoroughly. Next we transfer 1 ounce of the uniform mixture from glass B to glass A. The amount of soda in glass A at the end is

- A) 3.0 ounces
- B) 3.2 ounces
- C) 3.4 ounces
- D) 3.6 ounces
- E) 3.8 ounces

12. A computer engineer has designed a computer disk that stores information on two concentric rings, with the outer ring having a radius one and a half times the radius of the inner ring. Each ring holds the same amount of data per millimeter. If the total amount of data held by the two rings is 900,000 bytes, how much data does the inner ring hold?

- A) 360,000
- B) 540,000
- C) 600,000
- D) 674,000
- E) 675,000





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13. A 3-by-3 “checkerboard” contains a total of 14 squares – 9 1-by-1’s, 4 2-by-2’s and 1 3-by-3. How many squares does a 6-by-6 checkerboard contain?
- A) 28
 - B) 36
 - C) 42
 - D) 81
 - E) 91
14. Bob’s wristwatch, which does not distinguish between AM and PM, loses 5 minutes every hour. At 10 AM on August 1 his watch was accurate. When will his watch next be accurate?
- A) 10 AM August 6
 - B) 10 PM August 6
 - C) 10 AM August 7
 - D) 10 PM August 7
 - E) 10 AM August 8
15. If the size of a bacteria colony doubles every 5 hours, how long will it take for the number of bacteria to triple?
- A) less than 5 hours
 - B) between 5 and 7.5 hours
 - C) exactly 7.5 hours
 - D) between 7.5 and 10 hours
 - E) more than 10 hours

16. Your new long distance calling plan has a monthly fee of \$4.95. Additionally, for each call you get charged 10 cents a minute for the first 5 minutes and then only 4 cents a minute thereafter. If you made 7 calls for a total of 255 minutes last month, and the only call that lasted under 5 minutes was three minutes long, how much is your long distance phone bill?
17. Graphic Designs, Inc. is painting a logo for a new company. The logo is an equilateral triangle with an inscribed circle. Graphic Designs has already constructed the outer triangle, and now would like to construct the inner circle. If the sides of the triangle each measure 2 ft., what is the radius of the circle?

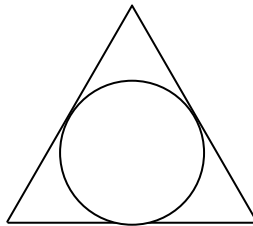
A) $\frac{\sqrt{3}}{3}$ ft.

B) $\sqrt{\frac{2}{3}}$ ft.

C) $\frac{\sqrt{3}}{2}$ ft.

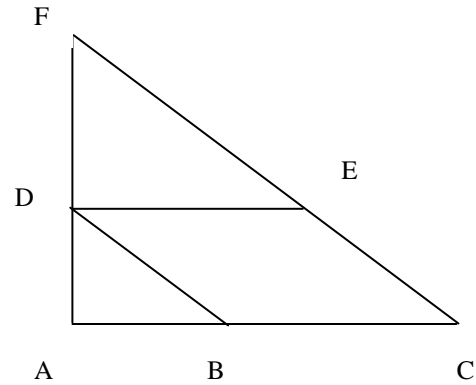
D) 1 ft.

E) $\frac{2}{3}\pi$ ft.



18. In the accompanying figure, triangle CAF is a right triangle, quadrilateral BCED is a parallelogram, segment AC has length 20, segment DE has length 12, and segment EF has length 15. What is the length of segment AD?

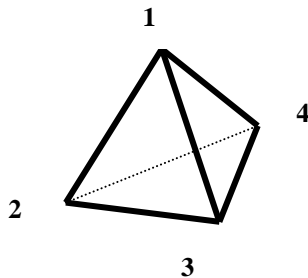
- A) 4
- B) 6
- C) 8
- D) 9
- E) 10



19. Poles of heights a and b stand distance d apart on level ground. Guy wires are strung from the top of each pole to the base of the other. The height above the ground of the point where the wires cross is

- A) $\frac{d}{2}$
- B) $\frac{a+b}{2}$
- C) $\frac{ab}{a+b}$
- D) $\frac{a-b}{2}$
- E) \sqrt{ab}

20. Positive integers a, b, c and d satisfy $a = b^2, c = d^2$ and $a - c = 7$. Then $b + d =$
- A) 7
B) 9
C) 11
D) 13
E) 25
21. A parking meter contains only quarters and dimes; there are 3 more quarters than dimes. If the amount inside is a whole number of dollars, what is the smallest possible total amount?
- A) \$5.00
B) \$6.00
C) \$8.00
D) \$12.00
E) \$35.00
22. Eric has a small tetrahedron on his desk. He picks it up and fidgets with it for a while, then sets it back down. To the casual observer, the tetrahedron now appears to be exactly as it was before Eric picked it up. However, Eric has marked the four vertices so he can tell them apart, and he set the tetrahedron down so that no vertex is in its original position. How many different ways could Eric do this?



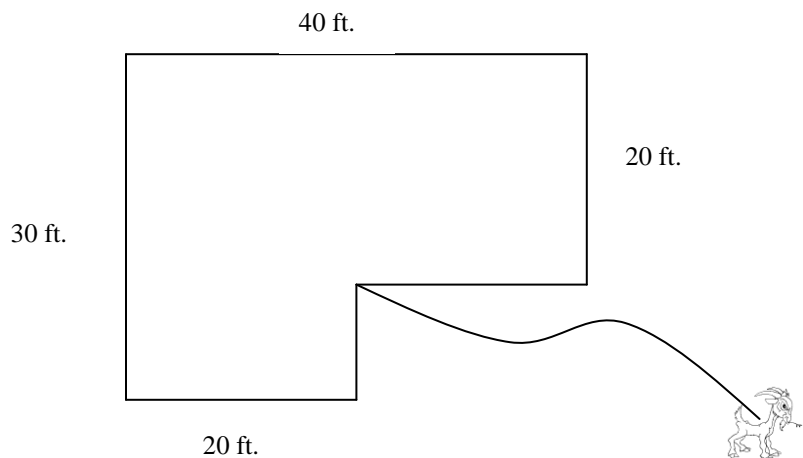
- A) 2
B) 3
C) 6
D) 9
E) 11

23. Fred, Dan, and Michelle decide to race for 200 miles. Dan drives the 200 miles at a constant speed of 50 mph. Fred drives the first 100 miles at 40 mph and the last 100 miles at 60 mph. Michelle drives the first 100 miles at 70 mph and the last 100 miles at 30 mph. Who wins the race?

- A) Michelle
- B) Fred
- C) Dan
- D) It's a tie
- E) There is not enough information to determine the winner

24. A goat is chained to the corner of a building with a 30 foot chain as pictured. All angles are right angles and the dimensions are given in the picture. What area can the goat graze?

- A) 15π sq. ft.
- B) 60π sq. ft.
- C) 350π sq. ft.
- D) 700π sq. ft.
- E) 1400π sq. ft.



25. Five students have access to a chat room. Given the following information, who is chatting? Either Kevin or Heather, or both, are chatting. Either Randy or Vijay, but not both, are chatting. If Abby is chatting, so is Randy. Vijay and Kevin are either both chatting or neither is. If Heather is chatting, then so are Abby and Kevin.
- A) Abby and Randy
 - B) Heather, Abby, and Kevin
 - C) Vijay and Abby
 - D) Kevin and Vijay
 - E) All of them are chatting
26. If the center of an equilateral triangle of side length 1 is moved 5 units, then no vertex of the triangle can move more than
- A) $5 - \sqrt{3}$ units
 - B) $5 + \frac{2}{\sqrt{3}}$ units
 - C) $5 + \sqrt{3}$ units
 - D) $5 + \frac{\sqrt{3}}{2}$ units
 - E) 5 units
27. An equilateral triangle and a regular hexagon have equal perimeters. The triangle has area A . The area of the hexagon is
- A) $\frac{2}{3}A$
 - B) $\frac{1}{2}A$
 - C) A
 - D) $\frac{3}{2}A$
 - E) $2A$

28. Consider a circle of radius r with a square inscribed in it. A triangle is then drawn from the midpoint of the side at the top to the bottom vertices. The area of the circle divided by the area of the triangle is:

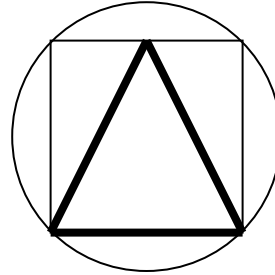
A) 2π

B) π

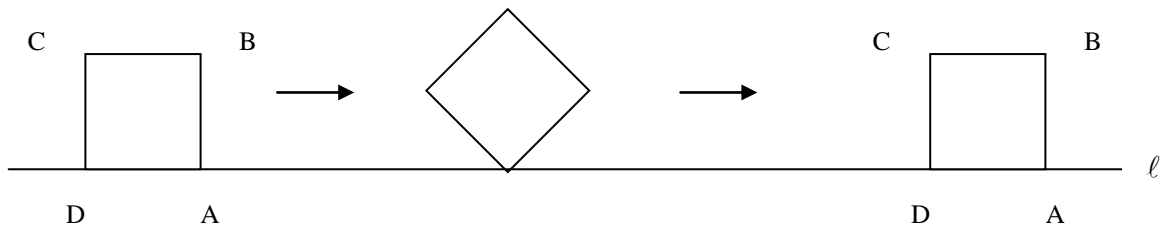
C) 3

D) 2

E) Impossible to compute



29. A square wheel of side length 1 is “rolled” left-to-right along the line ℓ , pivoting first on corner A, then B, then C and finally D, so that it is returned to its original orientation. How long is the path traced out by point D?



A) 4

B) $\frac{(2 + \sqrt{2})\pi}{2}$

C) $\frac{(3 + \sqrt{2})\pi}{2}$

D) $\frac{(1 + \sqrt{2})\pi}{2}$

E) 2π



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30. The sum of the prime divisors of $6^4 - 1 = 1295$ is

- A) 33
- B) 49
- C) 55
- D) 64
- E) 70