L-Systems
(added Summer, 2011)

From Wikipedia:
“An L-system or Lindenmayer system is a parallel rewriting system, namely a variant of a formal grammar, most famously used to model the growth processes of plant development, but also able to model the morphology of a variety of organisms. L-systems can also be used to generate self-similar fractals such as iterated function systems. L-systems were introduced and developed in 1968 by the Hungarian theoretical biologist and botanist from the University of Utrecht, Aristid Lindenmayer (1925 – 1989).”

The basic elements of an L-system form the alphabet. In the program L-Systems, the following symbols can be used as part of the alphabet:

- F, G draw a line the specified length and direction
- X, Y do nothing (these symbols are merely placeholders)
- A, B move (but do not draw) the initial point the specified length and direction. This is like drawing an invisible line.
- + rotate the current direction counterclockwise by the specified number of degrees.
- - rotate the current direction clockwise by the specified number of degrees.
- [ save the current values of position and angle for later use
- ] restore the position and angle for the saved values of the corresponding “[“.

L-systems create a sequence of images, where the definition of each image is dependent on the previously created image. As such, there needs to be a starting image, defined by a starting string. Often that starting string is “F”, which means that the starting image is a single line. A list of rules is then used to develop strings that will determine succeeding images. For example the rule “F -> G-F-G” means to replace every occurrence of the symbol “F” in the current string by the string “G-F-G”. If there is no rule for a symbol, that symbol is left alone when defining the next string.

The following parameters are used to determine how to start the drawing process:

- xstart the x-coordinate of the initial endpoint of the first line
- ystart the y-coordinate of the initial endpoint of the first line.
- length the length of all lines drawn in the initial image
- scale the factor by which the lengths of all lines are scaled down in the next image.
- angle the direction in which the initial line is drawn. The line is drawn in the direction of the terminal side of an angle in standard position, that is, an angle whose initial side is directed to the right and whose terminal side is rotated counterclockwise from the initial side “angle” degrees.
- degrees the number of degrees to rotate counterclockwise (if the symbol “+” is encountered) or clockwise (if the symbol “-“ is encountered).

So for example, suppose the initial string is “F” with rule “F -> F+F-F-F+F”. Also suppose that xstart = 0.125, ystart = .25, length = .75, scale = 1/3, angle = 0 and degrees = 90. Figure 1 shows the initial image along with the next eight images in the sequence. Each square represents
0 ≤ x ≤ 1, 0 ≤ y ≤ 1. Note that the first iteration after the initial one corresponds to the string “F+F-F-F+F”, whose image is shown in the top row, middle square. The next iteration after that corresponds to the string

“F+F-F+F + F+F-F+F – F+F-F-F+F – F+F-F-F+F + F+F-F-F+F”

whose image is shown in the top row, right square. The sequence of images being created converges to a variant of the Koch curve.

Now let’s take a look at the program **L-Systems**. After the title pages are displayed, your screen should look like Figure 2. At the right is a window with a green border. This is the status
window, which lists all of the parameters that will be used to create the next sequence of images. At the top is a list of pre-defined L-systems. They are:

- Koch Snowflake
- Levy C Curve
- Sierpinski Triangle
- Dragon Curve
- Space Filling Curve
- Plant

At the bottom of the list is the “Home Made” choice, which we will discuss later.

Below the list of pre-defined images are the graph mode options. The default one is the “Tic Tac Toe” option in which a 3 x 3 grid of images is formed. The “Full Screen” option utilizes the full graph window on the left at each iteration, which gives larger images with higher resolution. The text boxes below the graph mode define the initial string, the parameters, and the rules for the currently defined L-System. These text boxes are disabled for editing when any of the pre-defined L-systems are chosen.

When the “Go!” button is pressed, the image corresponding to the initial string (the initial state) is drawn. When in “Tic Tac Toe” graphing mode, this image is drawn in the left square, top row. The “Go!” button is replaced by two buttons, a “Next” button and a “Cancel” button. Also a window at the bottom appears that indicates which iteration is being drawn. When in “Tic Tac Toe” graphing mode only, after the eighth iteration is drawn, that is, when all of the boxes are filled in, the “Next” and “Cancel” boxes are replaced by “Repeat” and “Stop” buttons.
Figure 3 shows the dragon curve graphed in full screen mode at iteration 15.

Figure 3

When “Home Made” is selected, all of the text boxes defining the parameters and rules for the current L-system are enabled for editing. What follows are a few L-system images using the “Home Made” option.

Figure 4 depicts the initial image and the first four iterations of the L-system with initial string “F+XF+F+XF”, rule “X -> XF-F+F-XF+XF-F+X’” and parameters xstart = .5, ystart = .125, length = .25, scale = .48, angle = 0 and degrees = 90.

Figure 5 depicts the initial image and the first five iterations of the L-system with initial string “F+F+F+F”, rule “F -> FF+F+F+FF” and parameters xstart = .125, ystart = .125, length = .75, scale = 1/3, angle = 0 and degrees = 90.

Figure 6 is a depiction of a southern India design called the Anklets of Krishna. It is the sixth iteration of the L-system with initial string “-X—X”, rule “X -> XFX—XFX”, xstart = .5, ystart = .875, length = .75, scale = .48, angle = 0, and degrees = 45.
The last example demonstrates a bracketed L-system. The initial string is “F” with rule “FF[+F][−F]”. xstart = .5, ystart = 0.125, length = .375, scale = .5, angle = 90, and degrees = 45. The left bracket means to save the current position and angle for future use. When the corresponding right bracket is encountered go back to that saved position and angle then proceed from there. Note that during iteration 1 (middle graph top row of Figure 7), two vertical lines are drawn one on top of the other, then the third line is drawn on top of the second line but at an angle 45 degrees counterclockwise from vertical. The first set of brackets force the fourth line to be drawn, not at the end of the third line, but at the end of the second line, this time at an angle of 45 degrees clockwise from vertical. Figure 7 shows the generation of the first 8 iterations of this L-system.
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