Activity 8.6 – Integration by Substitution

1. (a) $\int e^{4x+5}dx = \int e^u \cdot \frac{1}{4}du = \frac{1}{4}\int e^u du = \frac{1}{4}e^u + C = \frac{1}{4}e^{4x+5} + C$

$u = 4x + 5$; $du = 4dx$; $dx = \frac{1}{4}du$

(b) $\int x^2 e^{x^3} dx = \int e^u \cdot \frac{1}{3}du = \frac{1}{3}\int e^u du = \frac{1}{3}e^u + C = \frac{1}{3}e^{x^3} + C$

$u = x^3$; $du = 3x^2dx$; $x^2dx = \frac{1}{3}du$

(c) $\int \frac{1}{9t+4}dt = \int \frac{1}{u}\cdot\frac{1}{9}du = \frac{1}{9}\int \frac{1}{u}du = \frac{1}{9}\ln|u| + C = \frac{1}{9}\ln|9t+4| + C$

$u = 9t + 4$; $du = 9dt$; $dt = \frac{1}{9}du$

(d) $\int \frac{x+6}{x^2+12x+10}dx = \int \frac{1}{u}\cdot\frac{1}{2}du = \frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+12x+10| + C$

$u = x^2 + 12x + 10$; $du = 2x + 12 = 2(x + 6)$; $(x + 6)dx = \frac{1}{2}du$

(e) $\int \sin^5\theta\ \cos\theta\, d\theta = \int u^5 du = \frac{1}{6}u^6 + C = \frac{1}{6}\sin^6\theta + C$

$u = \sin\theta$; $du = \cos\theta\, d\theta$

2. (a) $\int_0^1 x^3\left(1+2x^4\right)^3 dx = \int_1^3 u^3\cdot\frac{1}{8}du = \frac{1}{8}\int_1^3 u^3 du = \frac{u^4}{32}\Big|_1^3 = \frac{81}{32} - \frac{1}{32} = \frac{80}{32} = \frac{5}{2}$

$u = 1 + 2x^4$; $du = 8x^3dx$; $x^3\,dx = \frac{1}{8}du$

$x = 0 \rightarrow u = 1 + 2(0)^4 = 1$; $\quad x = 1 \rightarrow u = 1 + 2(1)^4 = 3$

(b) $\int_1^2 x\cdot\sqrt[3]{5-x^2}\,dx = -\frac{1}{2}\int_4^1 \sqrt[3]{u}\,du = \left(-\frac{3}{8}u^{4/3}\right)\Big|_4^1 = \left(-\frac{3}{8}\cdot 1^{4/3}\right) - \left(-\frac{3}{8}\cdot 4^{4/3}\right) = -\frac{3}{8} + \frac{3\sqrt[3]{4}}{2}$

$u = 5 - x^2$; $du = -2xdx$; $xdx = -\frac{1}{2}du$

$x = 1 \rightarrow u = 5 - (1)^2 = 4$; $\quad x = 2 \rightarrow u = 5 - (2)^2 = 1$

(c) $\int_0^{\pi/4} e^{\sin(2x)}\cos(2x)\,dx = \frac{1}{2}\int_0^1 e^u du = \left(\frac{1}{2}e^u\right)\Big|_0^1 = \left(\frac{1}{2}\cdot e^1\right) - \left(\frac{1}{2}\cdot e^0\right) = \frac{e}{2} - \frac{1}{2}$

$u = \sin(2x)$; $du = 2\cos(2x)dx$; $\cos(2x)dx = \frac{1}{2}du$

$x = 0 \rightarrow u = \sin(0) = 0$; $\quad x = \pi/4 \rightarrow u = \sin(\pi/2) = 1$

3. (a) $\int \frac{3x^7}{\left(x^8+1\right)^2}dx = \frac{3}{8}\int \frac{1}{u^2}du = -\frac{3}{8u} + C = -\frac{3}{8x^8+8} + C$; $\int_0^1 \frac{3x^7}{\left(x^8+1\right)^2}dx = \left(-\frac{3}{8x^8+8}\right)\Big|_0^1 = \frac{3}{16}$

$u = x^8 + 1$; $du = 8x^7dx$; $3x^7dx = \frac{3}{8}du$

(b) $\int \frac{(\ln x)^2}{x}dx = \int u^2 du = \frac{1}{3}u^3 + C = \frac{1}{3}(\ln x)^3 + C$; $\int_1^3 \frac{(\ln x)^2}{x}dx = \frac{1}{3}(\ln x)^3\Big|_1^3 = \frac{1}{3}(\ln 3)^3$

$u = \ln x$; $du = \frac{1}{x}dx$