Examples 8.5 – The Fundamental Theorem of Calculus (Part 2)

1. Let $F(x) = \int_0^x \arctan\left(t^3\right)dt$. Evaluate each of the following.

   (a) $F(0)$ (b) $F'(x)$ (c) $F'(1)$

Solution: (a) $F(0) = \int_0^0 \arctan\left(t^3\right)dt = 0$

(b) $F'(x) = \dfrac{d}{dx}\left(\int_0^x \arctan\left(t^3\right)dt\right) = \arctan\left(x^3\right)$

(c) $F'(1) = \arctan\left(1^3\right) = \dfrac{\pi}{4}$

2. Find the derivative (with respect to x) of each function.

   (a) $f(x) = \int_{-1}^x \left(\frac{1}{3}t^9 - 4t^5\right)^{12} dt$ (b) $g(x) = \int_x^1 \left(\dfrac{1-\sin t}{2+\cos t}\right)dt$ (c) $h(x) = \int_0^{2x^3} \ln(t+1)\, dt$

Solution: (a) $f'(x) = \dfrac{d}{dx}\left(\int_{-1}^x \left(\frac{1}{3}t^9 - 4t^5\right)^{12} dt\right) = \left(\frac{1}{3}x^9 - 4x^5\right)^{12}$

(b) $g(x) = \int_x^1 \left(\dfrac{1-\sin t}{2+\cos t}\right)dt = -\int_1^x \left(\dfrac{1-\sin t}{2+\cos t}\right)dt$, so $g'(x) = -\dfrac{1-\sin x}{2+\cos x}$

(c) $h'(x) = \dfrac{d}{dx}\left(\int_0^{2x^3} \ln(t+1)\, dt\right) = \ln\left(2x^3+1\right)\cdot 6x^2$

3. Let $F(x) = \int_0^{-x} (t+1)e^t dt$. Find F' and F''.

Solution: $F'(x) = \dfrac{d}{dx}\left(\int_0^{-x} (t+1)e^t dt\right) = (-x+1)e^{-x}\cdot(-1) = (x-1)e^{-x}$

$F''(x) = \dfrac{d}{dx}\left((x-1)e^{-x}\right) = (1)e^{-x} + (x-1)\left(-e^{-x}\right) = (2-x)e^{-x}$