Activity 8.5 – The Fundamental Theorem of Calculus (Part 2)

1. (a) $f'(x) = \sqrt{x^5 + 5x^2}$

   (b) $y(x) = -\int_1^x \frac{t^3}{e^t + 4} dt$, so $y'(x) = -\frac{x^3}{e^x + 4}$

   (c) $F'(x) = \arctan((2x)^2 + 10) \cdot 2 = 2\arctan(4x^2 + 10)$

   (d) $H'(x) = \frac{1}{\sqrt[3]{\left(e^x\right)^2 + 2e^x}} \cdot e^x = \frac{e^x}{\sqrt[3]{e^{2x} + 2e^x}}$

   (e) $g(x) = -\int_1^{5x^2} \cos(\ln(t))dt$, so $g'(x) = -\cos\left(\ln\left(5x^2\right)\right) \cdot 10x = -10x\cos\left(\ln\left(5x^2\right)\right)$

2. (a) $G\left(\frac{1}{3}\right) = \int_1^1 e^{-t^2} dt = 0$

   (b) $G'(x) = e^{-(3x)^2} \cdot 3 = 3e^{-9x^2}$

   (c) $G'(0) = 3e^{-9(0)^2} = 3$

3. (a) Set $F'(x) = \left(x^2 - 1\right)e^{x^2} \cdot 2x = 0$ to get $x = -1, 0, 1$. (Note that $e^{x^2} \neq 0$.)

   (b) Rewrite $F'(x) = \left(2x^3 - 2x\right)e^{x^2}$. Set

$$
\begin{aligned}
F''(x) &= \left(6x^2 - 2\right)e^{x^2} + \left(2x^3 - 2x\right)e^{x^2} \cdot 2x \\
&= 2\left(2x^4 + x^2 - 1\right)e^{x^2} \\
&= 2\left(2x^2 - 1\right)\left(x^2 + 1\right)e^{x^2} \\
&= 0
\end{aligned}
$$

   to get $2x^2 - 1 = 0$, or $x = \pm\sqrt{\frac{1}{2}}$ (Note that $e^{x^2} \neq 0$ and $x^2 + 1 \neq 0$.) A sign test shows that these are the locations of the inflection points: