## Activity 8.4 – The Fundamental Theorem of Calculus (Part 1)

1. (a) $\int_1^{\sqrt{2}} \frac{8}{1+x^2}\,dx = 8\arctan(x)\Big|_1^{\sqrt{2}} = 8\arctan\left(\sqrt{2}\right) - 8\arctan(1) \approx 1.3593$

(b) $\int_1^2 \frac{4}{x^3}\,dx = -\frac{2}{x^2}\Big|_1^2 = \left(-\frac{2}{4}\right) - \left(-\frac{2}{1}\right) = \frac{3}{2}$

(c) $\int_0^2 \frac{4}{x^3}\,dx$ The FTC cannot be used since the integrand is not continuous at $x = 0$.

(d) $\int_0^3 \frac{1}{x-2}\,dx$ The FTC cannot be used since the integrand is not continuous at $x = 2$.

(e) $\int_4^9 6\sqrt{x}\,dx = \frac{6x^{3/2}}{3/2}\Big|_4^9 = 4(9)^{3/2} - 4(4)^{3/2} = 76$

(f) $\int_0^{\pi} 3\cos x\,dx = 3\sin x\Big|_0^{\pi} = 3\sin\pi - 3\sin 0 = 0$

(g) $\int_{-1}^1 e^{5x}\,dx = \frac{1}{5}e^{5x}\Big|_{-1}^1 = \frac{1}{5}e^5 - \frac{1}{5}e^{-5} \approx 29.681$

(h) $\int_0^{0.5} \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x\Big|_0^{0.5} = \arcsin(0.5) - \arcsin(0) = \frac{\pi}{6}$

2. (a) NOTE THAT THE FOLLOWING COMPUTATION IS INCORRECT!

$$\int_{-1}^1 \frac{1}{x^2}dx = \int_{-1}^1 x^{-2}dx = -\frac{1}{x}\Big|_{-1}^1 = (-1) - (1) = -2$$

(b) The graph is above the $x$-axis on $[-1, 1]$, so the net area is positive, not negative.

3. (a) Set $\int_0^3 x^2dx = \left(t^*\right)^2(3-0)$ so that $\frac{1}{3}x^3\Big|_0^3 = 3\left(t^*\right)^2$.

Therefore, $9 = 3\left(t^*\right)^2$, and so $t^* = \sqrt{3} \approx 1.732$.

(b) $(t^*, f(t^*))$

(c) $\int_a^b f(x)dx$

(d) $f(t^*)(b-a)$