Activity 8.4[1†‡] – The Fundamental Theorem of Calculus (Part 1)

FOR DISCUSSION: *State Part 1 of the Fundamental Theorem of Calculus in your own words. Geometrically, what does Part 1 of the FTC measure?*

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1. If the given integral satisfies the hypothesis for Part 1 of the FTC, then compute it. Otherwise, explain why the FTC cannot be used. (**HINT**: Consider continuity.)

   (a) $\int_{1}^{\sqrt{2}} \frac{8}{1+x^2}\,dx =$

   (b) $\int_{1}^{2} \frac{4}{x^3}\,dx =$

   (c) $\int_{0}^{2} \frac{4}{x^3}\,dx =$

   (d) $\int_{0}^{3} \frac{1}{x-2}\,dx =$

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[1] This activity contains new material.

[†] This activity is referenced in Examples 8.4 and Lesson 8.5.

[‡] This activity has supplemental exercises.

(e) $\int_4^9 6\sqrt{x}dx =$

(f) $\int_0^{\pi} 3\cos x dx =$

(g) $\int_{-1}^{1} e^{5x} dx =$

(h) $\int_0^{0.5} \frac{1}{\sqrt{1-x^2}} dx =$

2. The FTC cannot be used to compute $\int_{-1}^{1} \frac{1}{x^2}dx$ since the integrand is not continuous on [–1, 1].

   (a) Suppose we forgot to check for continuity. Use the FTC to evaluate the integral. You will get an answer…

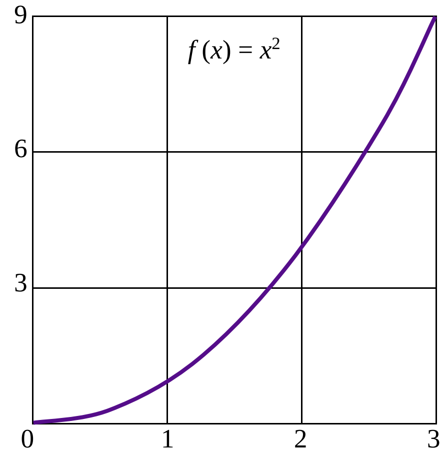
   $\int_{-1}^{1} \frac{1}{x^2}dx$

   (b) View or sketch the graph of $y = \frac{1}{x^2}$ on [–1, 1]. Explain why your answer from Part (a) must be incorrect.

3. (**OPTIONAL**) The **Mean Value Theorem for Integrals** states that if f is continuous on $[a, b]$, then there is at least one point t^* in $[a, b]$ such that $\int_a^b f(x)dx = f(t^*)(b-a)$. (proof below)

(a) Let $f(x) = x^2$ on [0, 3]. Find t^* in [0, 3] such that $\int_a^b f(x)dx = f(t^*)(b-a)$.

(b) Mark the point $(t^*, f(t^*))$ on the graph of $f(x) = x^2$.



(c) In terms of area, what does $\int_a^b f(x)dx$ represent? Shade this area on the graph.

(d) In terms of area, what does $f(t^*)(b-a)$ represent? Shade this area on the graph.

(e) Imagine the area bounded by f "settling down" into the shape of a rectangle. The MVT for Integrals helps us to find the height $f(t^*)$ of the rectangle.

Proof: Suppose that f is continuous on $[a, b]$. By the EVT, f assumes a minimum value m and a maximum value M on $[a, b]$. Then for all x in $[a, b]$,

$$m \le f(x) \le M$$

$$\int_a^b m\,dx \le \int_a^b f(x)\,dx \le \int_a^b M\,dx$$

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$

$$m \le \tfrac{1}{b-a}\int_a^b f(x)\,dx \le M$$

By the IVT, f attains all values between m and M, including $\frac{1}{b-a}\int_a^b f(x)\,dx$. Therefore, there is a t^* in $[a, b]$ such that $f(t^*) = \frac{1}{b-a}\int_a^b f(x)\,dx$. It follows that $\int_a^b f(x)dx = f(t^*)(b-a)$. ■