## Activity 8.3 – Rolle’s Theorem and the Mean Value Theorem

1. (a) Set $f'(c) = 3c^2 - 1 = \frac{f(2) - f(-2)}{2-(-2)} = \frac{8-(-4)}{4} = 3$ to get $c = \pm\sqrt{\frac{4}{3}}$ in the interval (–2, 2).

   (b) Set $g'(c) = -\frac{1}{c^2} = \frac{g(5)-g(3)}{5-3} = \frac{\frac{1}{5}-\frac{1}{3}}{2} = -\frac{1}{15}$ to get $c = \pm\sqrt{15}$ , but only $\sqrt{15}$ is in (3, 5).

2. (a) Since *f* has a vertical asymptote at *x* = 0, it is not continuous on [0,4]. Since it is not continuous at *x* = 0, it is not differentiable at *x* = 0.

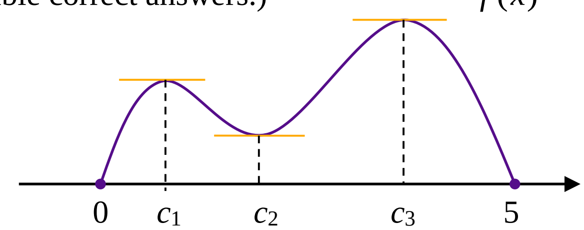
   (b) Since *g* only has one discontinuity at *x* = 6, it is continuous on [0, 4]. Since *g′* is undefined only at *x* = 6 (use the quotient rule), *g* is differentiable everywhere except at *x* = 6. In particular, it is differentiable on [0, 4].

   (c) Since *h* has a vertical asymptote at *x* = –3, it is not continuous on [–4 ,5]. Since it is not continuous at *x* = –3, it is not differentiable at *x* = –3.

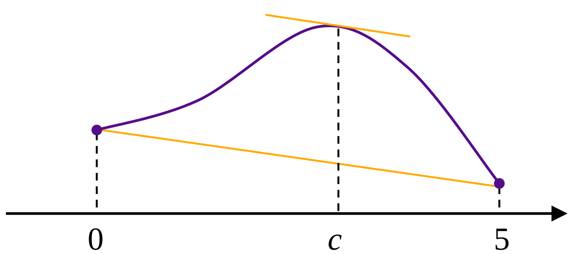
   (d) Since $F(x) = \begin{cases} -x^2 + x + 6, & \text{if } x < 3 \\ x^2 - x - 6, & \text{if } x \geq 3 \end{cases}$, it follows that $F'(x) = \begin{cases} -2x + 1, & \text{if } x < 3 \\ 2x - 1, & \text{if } x > 3 \end{cases}$.

   From the left of 3, *F* has a slope of –5, and from the right of 3, *F* has a slope of 5. Therefore, *F* is not differentiable at *x* = 3 (graph has a corner), and hence it is not differentiable on [1, 4]. It is continuous everywhere, however, as is seen by substituting *x* = 3 into the piecewise formulas for *F*.

3. (There are many possible correct answers.)




4. (There are many possible correct answers.)




5. Your average velocity between 8:00 and 8:03 is $\frac{\Delta\text{position}}{\Delta\text{time}} = \frac{4\text{ miles}}{3\text{ minutes}} \times \frac{60\text{ minutes}}{1\text{ hour}} = 80\text{ mi/h}$ .

   By the MVT, there exists a time between 8:00 and 8:03 at which your instantaneous velocity was equal to your average velocity. In other words, you are guilty of traveling at a speed of 80 mi/h during this time interval.