



Activity 8.3 – Rolle’s Theorem and the Mean Value Theorem

FOR DISCUSSION: Describe what it means for f to be continuous on the interval $[a, b]$.
Describe what it means for f to be differentiable on the interval $[a, b]$.

1. (a) Let $f(x) = x^3 - x + 2$. Find all numbers c in $(-2, 2)$ such that $f'(c) = \frac{f(2) - f(-2)}{2 - (-2)}$.

(b) Let $g(x) = \frac{1}{x}$. Find all numbers c in $(3, 5)$ such that $g'(c) = \frac{g(5) - g(3)}{5 - 3}$.

2. For each function, decide if it is continuous on the given interval. Then decide if it is differentiable on the given interval. Justify your answers.

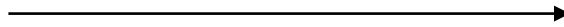
(a) $f(x) = \frac{x-1}{x^2+2x}$ on $[0, 4]$.

(b) $g(x) = \frac{x^2-1}{x-6}$ on $[0, 4]$.

(c) $h(x) = \frac{x+1}{x+3}$ on $[-4, 5]$.

(d) $F(x) = |x^2 - x - 6|$ on $[1, 4]$.

3. Sketch the graph of a function f that is continuous on $[0, 5]$, differentiable on $(0, 5)$, and such that $f(0) = f(5) = 0$. Mark all values of c that satisfy Rolle's Theorem.



4. Sketch the graph of a function g that is continuous on $[0, 5]$, differentiable on $(0, 5)$, and such that $g(0) \neq 0$ and $g(5) \neq 0$. Mark all values of c that satisfy the Mean Value Theorem.



5. (**OPTIONAL**) Suppose you are driving westbound along I-86 in western New York, where the posted speed limit is 65 mi/h. As you pass the Hornell exit at 8:00 a.m., a police cruiser records your speed as 65 mi/h. Just before the Almond exit, four miles down the road, a second cruiser records your speed as 60 mi/h at 8:03 a.m. Explain why you are guilty of speeding.