Activity 8.3 – Rolle's Theorem and the Mean Value Theorem

FOR DISCUSSION: *Describe what it means for f to be continuous on the interval [a, b].*
Describe what it means for f to be differentiable on the interval [a, b].

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1. (a) Let $f(x) = x^3 - x + 2$. Find all numbers c in (–2, 2) such that $f'(c) = \dfrac{f(2) - f(-2)}{2 - (-2)}$.

   (b) Let. $g(x) = \dfrac{1}{x}$. Find all numbers c in (3, 5) such that $g'(c) = \dfrac{g(5) - g(3)}{5 - 3}$.

2. For each function, decide if it is continuous on the given interval. Then decide if it is differentiable on the given interval. Justify your answers.

   (a) $f(x) = \dfrac{x - 1}{x^2 + 2x}$ on [0, 4].

   (b) $g(x) = \dfrac{x^2 - 1}{x - 6}$ on [0, 4].

   (c) $h(x) = \dfrac{x + 1}{x + 3}$ on [–4, 5].

(d) $F(x) = \left| x^2 - x - 6 \right|$ on [1, 4].

3. Sketch the graph of a function f that is continuous on [0, 5], differentiable on (0, 5), and such that $f(0) = f(5) = 0$. Mark all values of c that satisfy Rolle's Theorem.

4. Sketch the graph of a function g that is continuous on [0, 5], differentiable on (0, 5), and such that $g(0) \neq 0$ and $g(5) \neq 0$. Mark all values of c that satisfy the Mean Value Theorem.

5. (**OPTIONAL**) Suppose you are driving westbound along I-86 in western New York, where the posted speed limit is 65 mi/h. As you pass the Hornell exit at 8:00 a.m., a police cruiser records your speed as 65 mi/h. Just before the Almond exit, four miles down the road, a second cruiser records your speed as 60 mi/h at 8:03 a.m. Explain why you are guilty of speeding.