## Activity 8.2 – The Definition of Net Area

1. $a = 1$; $b = 2$; $\Delta x = \frac{2-1}{n} = \frac{1}{n}$; $x_k^* = 1 + \frac{1}{n} \cdot k$; $f\left(x_k^*\right) = 3\left(1 + \frac{1}{n} \cdot k\right)^2 = 3 + \frac{6}{n} \cdot k + \frac{3}{n^2} \cdot k^2$;

$$f\left(x_k^*\right)\Delta x = \left(3 + \frac{6}{n} \cdot k + \frac{3}{n^2} \cdot k^2\right)\left(\frac{1}{n}\right) = \frac{3}{n} + \frac{6}{n^2} \cdot k + \frac{3}{n^3} \cdot k^2$$

$$\sum_{k=1}^{n} f\left(x_k^*\right)\Delta x = \sum_{k=1}^{n}\left(\frac{3}{n} + \frac{6}{n^2} \cdot k + \frac{3}{n^3} \cdot k^2\right) = \frac{3}{n}\sum_{k=1}^{n} 1 + \frac{6}{n^2}\sum_{k=1}^{n} k + \frac{3}{n^3}\sum_{k=1}^{n} k^2$$

$$= \frac{3}{n}(n) + \frac{6}{n^2}\left(\frac{n(n+1)}{2}\right) + \frac{3}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$= 3 + \frac{3(n+1)}{n} + \frac{(n+1)(2n+1)}{2n^2}$$

$$\int_1^2 3x^2\, dx = \lim_{n\to+\infty}\sum_{k=1}^{n} f\left(x_k^*\right)\Delta x = \lim_{n\to+\infty}\left(3 + \frac{3(n+1)}{n} + \frac{(n+1)(2n+1)}{2n^2}\right) = 3 + 3 + 1 = 7$$

2. $a = 0$; $b = 3$; $\Delta x = \frac{3-0}{n} = \frac{3}{n}$; $x_k^* = \frac{3}{n} \cdot k$; $f\left(x_k^*\right) = 2\left(\frac{3}{n} \cdot k\right)^2 - \left(\frac{3}{n} \cdot k\right) = \frac{18}{n^2} \cdot k^2 - \frac{3}{n} \cdot k$;

$$f\left(x_k^*\right)\Delta x = \left(\frac{18}{n^2} \cdot k^2 - \frac{3}{n} \cdot k\right)\left(\frac{3}{n}\right) = \frac{54}{n^3} \cdot k^2 - \frac{9}{n^2} \cdot k$$

$$\sum_{k=1}^{n} f\left(x_k^*\right)\Delta x = \sum_{k=1}^{n}\left(\frac{54}{n^3} \cdot k^2 - \frac{9}{n^2} \cdot k\right) = \frac{54}{n^3}\sum_{k=1}^{n} k^2 - \frac{9}{n^2}\sum_{k=1}^{n} k = \frac{54}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) - \frac{9}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$$= \frac{9(n+1)(2n+1)}{n^2} - \frac{9(n+1)}{2n}$$

$$\int_1^2 3x^2\, dx = \lim_{n\to+\infty}\sum_{k=1}^{n} f\left(x_k^*\right)\Delta x = \lim_{n\to+\infty}\left(\frac{9(n+1)(2n+1)}{n^2} - \frac{9(n+1)}{2n}\right) = 18 - \frac{9}{2} = \frac{27}{2}$$

3. $\int_0^5 \left|t^2 - 2t\right| dt = -\int_0^2 \left(t^2 - 2t\right)dt + \int_2^5 \left(t^2 - 2t\right)dt = -\left(-\frac{4}{3}\right) + \left(\frac{54}{3}\right) = \frac{58}{3}$ m

4. (a) $\int_2^{-1} f(x)dx = -41$; (b) $\int_2^6 g(x)dx = -6$; (c) $\int_3^5 (4h(x) - 3)dx = 4\int_3^5 h(x)dx - \int_3^5 3dx = 18$

5. (a) $\int_a^a f(x)dx = \lim_{n\to+\infty}\sum_{k=1}^{n} f(x_k^*) \cdot \frac{a-a}{n} = 0$

(b) $\int_b^a f(x)dx = \lim_{n\to+\infty}\sum_{k=1}^{n} f(x_k^*) \cdot \frac{a-b}{n} = -\lim_{n\to+\infty}\sum_{k=1}^{n} f(x_k^*) \cdot \frac{b-a}{n} = -\int_a^b f(x)dx$

(c) $\int_a^b k \cdot f(x)dx = \lim_{n\to+\infty}\sum_{k=1}^{n} k \cdot f(x_k^*) \cdot \frac{b-a}{n} = k \cdot \lim_{n\to+\infty}\sum_{k=1}^{n} f(x_k^*) \cdot \frac{b-a}{n} = k \cdot \int_a^b f(x)dx$

(d) $\int_a^b \left(f(x) \pm g(x)\right)dx = \lim_{n\to+\infty}\sum_{k=1}^{n}\left(f(x_k^*) \pm g(x_k^*)\right) \cdot \frac{b-a}{n} = \lim_{n\to+\infty}\sum_{k=1}^{n} f(x_k^*) \cdot \frac{b-a}{n} \pm \lim_{n\to+\infty}\sum_{k=1}^{n} g(x_k^*) \cdot \frac{b-a}{n}$

$$= \int_a^b f(x)dx \pm \int_a^b g(x)dx$$