Activity 8.2[‡] – The Definition of Net Area

FOR DISCUSSION: *In your own words, explain the meaning of each symbol in the expression*

$$\lim_{n \to +\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x$$

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1. Use the limit definition of the definite integral with right-hand endpoints to evaluate $\int_1^2 3x^2 dx$ by following the sequence of computations. Simplify your work at each step.

   $a =$ _____ $b =$ _____ $\Delta x =$ _____ $x_k^* = a + \Delta x \cdot k =$ __________

   $f\left(x_k^*\right) =$

   $f\left(x_k^*\right)\Delta x =$

   $\sum_{k=1}^{n} f\left(x_k^*\right)\Delta x =$ (closed form)

   $\lim_{n \to +\infty} \sum_{k=1}^{n} f\left(x_k^*\right)\Delta x =$

   Therefore, $\int_1^2 3x^2\, dx =$

[‡] This activity has supplemental exercises.

2. Use the limit definition of the definite integral with right-hand endpoints to evaluate $\int_0^3 (2x^2 - x)dx$ by following the sequence of computations. Simplify your work at each step.

$a =$ ______ $b =$ ______ $\Delta x =$ ______ $x_k^* = a + \Delta x \cdot k =$ ____________

$f(x_k^*) =$

$f(x_k^*)\Delta x =$

$\sum_{k=1}^{n} f(x_k^*)\Delta x =$

$\lim_{n \to +\infty} \sum_{k=1}^{n} f(x_k^*)\Delta x =$

Therefore, $\int_0^3 (2x^2 - x)dx =$

3. The velocity (in m/s) of an object is given by $v(t) = t^2 - 2t$. Use the properties of the definite integral and the Fundamental Theorem to find the total distance traveled on [0, 5]. That is, compute $\int_0^5 |t^2 - 2t|\, dt$.

4. Practice the properties of the definite integral.

(a) If $\int_{-1}^{2} f(x)\, dx = 41$, then $\int_{2}^{-1} f(x)\, dx =$

(b) If $\int_{2}^{10} g(x)\, dx = 15$ and $\int_{6}^{10} g(x)\, dx = 21$, then $\int_{2}^{6} g(x)\, dx =$

(c) If $\int_{3}^{5} h(x)\, dx = 6$, then $\int_{3}^{5} (4h(x) - 3)\, dx =$

5. (**OPTIONAL**) Verify some of the properties of the definite integral stated in Lesson 8.2 using the properties of summations, the limit laws, and the definition of the definite integral:

$$\int_a^b f(x)dx = \lim_{n\to+\infty} \sum_{k=1}^{n} f(x_k^*) \cdot \frac{b-a}{n}$$

(a) Write $\int_a^a f(x)dx$ in limit form. Show that $\int_a^a f(x)dx = 0$.

(b) Write $\int_b^a f(x)dx$ in limit form. Show that $\int_b^a f(x)dx = -\int_a^b f(x)dx$.

(c) Write $\int_a^b k \cdot f(x)dx$ in limit form. Show that $\int_a^b k \cdot f(x)dx = k \cdot \int_a^b f(x)dx$.

(d) Write $\int_a^b \left(f(x) \pm g(x)\right)dx$ in limit form. Show that $\int_a^b \left(f(x) \pm g(x)\right)dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$

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