Lesson 8.1 – Sigma Notation and Summations

Recall, the computation of a net area using definite integration amounts to adding the areas of increasing numbers of approximating rectangles. **Sigma notation** is used for summations involving many terms. If $f(k)$ is a function of *k*, and if *m* and *n* are integers such that $m \le n$, then

$$\sum_{k=m}^{n} f(k) = f(m) + f(m+1) + \cdots + f(n)$$

The variable *k* is the **index of summation**. A **closed form** for a summation is a formula that does not contain sigma. A few well-known closed forms will prove to be very useful to us.

Summation formulas (closed forms): Note that *k* must start at 1 in each of these formulas.

1. $\sum_{k=1}^{n} 1 = \underbrace{1+1+\cdots+1}_{n \text{ times}} = n$ (sum of *n* ones)

2. $\sum_{k=1}^{n} k = 1+2+\cdots+n = \dfrac{n(n+1)}{2}$ (sum of the first *n* positive integers)

3. $\sum_{k=1}^{n} k^2 = 1^2+2^2+\cdots+n^2 = \dfrac{n(n+1)(2n+1)}{6}$ (sum of the first *n* positive squares)

4. $\sum_{k=1}^{n} k^3 = 1^3+2^3+\cdots+n^3 = \dfrac{n^2(n+1)^2}{4}$ (sum of the first *n* positive cubes)

Finite addition is *commutative* in the sense that we can rearrange the terms and still get the same result. (Surprisingly, this is not necessarily true for infinite sums! You will learn more about this in Calculus II.) We also know that multiplication *distributes* over addition, and we apply the reverse of this property whenever we factor out a constant multiple from each term of a sum. Since both the derivative and the integral also have these properties, they should be easy to remember. We summarize these important properties below.

Properties:

1. $\sum_{k=1}^{n} \left(f(k) \pm g(k)\right) = \sum_{k=1}^{n} f(k) \pm \sum_{k=1}^{n} g(k)$ (sum/difference rule)

2. $\sum_{k=1}^{n} c \cdot f(k) = c \cdot \sum_{k=1}^{n} f(k)$ (constant multiple rule)