Activity 8.1 – Sigma Notation and Summations

1. (a) 16; (b) 13

2. (a) $\sum_{k=1}^{n}\left(4k^3-2\right)=4\sum_{k=1}^{n}k^3-2\sum_{k=1}^{n}1=4\left(\frac{n^2(n+1)^2}{4}\right)-2(n)=n^2(n+1)^2-2n$

   (b) $\sum_{k=1}^{n}\left(1+4k+4k^2\right)=\sum_{k=1}^{n}1+4\sum_{k=1}^{n}k+4\sum_{k=1}^{n}k^2=n+4\left(\frac{n(n+1)}{2}\right)+4\left(\frac{n(n+1)(2n+1)}{6}\right)$

   $=n+2n(n+1)+\frac{2}{3}n(n+1)(2n+1)$

3. (a) $\sum_{k=1}^{63}\left(4k^3-2\right)=(63)^2(63+1)^2-2\cdot 63=16{,}256{,}898$

   (b) $\sum_{k=1}^{40}(1+2k)^2=40+2\cdot 40(40+1)+\frac{2}{3}\cdot 40(40+1)(2\cdot 40+1)=91{,}880$

4. (a) $\sum_{k=1}^{n}\left(2+\frac{1}{n}\cdot k\right)^2\left(\frac{1}{n}\right)=\sum_{k=1}^{n}\left(4+\frac{4}{n}\cdot k+\frac{1}{n^2}\cdot k^2\right)\left(\frac{1}{n}\right)$

   $=\sum_{k=1}^{n}\left(\frac{4}{n}\cdot 1+\frac{4}{n^2}\cdot k+\frac{1}{n^3}\cdot k^2\right)$

   $=\frac{4}{n}\sum_{k=1}^{n}1+\frac{4}{n^2}\sum_{k=1}^{n}k+\frac{1}{n^3}\sum_{k=1}^{n}k^2$

   $=\frac{4}{n}(n)+\frac{4}{n^2}\left(\frac{n(n+1)}{2}\right)+\frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$

   $=4+\frac{2(n+1)}{n}+\frac{(n+1)(2n+1)}{6n^2}$

   (b) $\sum_{k=1}^{n}3\left(-1+\frac{2}{n}\cdot k\right)^2\left(\frac{2}{n}\right)=\sum_{k=1}^{n}\left(\frac{6}{n}\cdot 1-\frac{24}{n^2}\cdot k+\frac{24}{n^3}\cdot k^2\right)$

   $=\frac{6}{n}(n)-\frac{24}{n^2}\left(\frac{n(n+1)}{2}\right)+\frac{24}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$

   $=6-\frac{12(n+1)}{n}+\frac{4(n+1)(2n+1)}{n^2}$

5. (a) $\sum_{k=1}^{3}\left(F(x_k)-F(x_{k-1})\right)=F(x_1)-F(x_0)+F(x_2)-F(x_1)+F(x_3)-F(x_2)$

   $=F(x_3)-F(x_0)$

   (b) $\sum_{k=1}^{4}\left(F(x_k)-F(x_{k-1})\right)=F(x_1)-F(x_0)+F(x_2)-F(x_1)+F(x_3)-F(x_2)+F(x_4)-F(x_3)$

   $=F(x_4)-F(x_0)$

   (c) $\sum_{k=1}^{n}\left(F(x_k)-F(x_{k-1})\right)=F(x_n)-F(x_0)$