Activity 8.1[1†] – Sigma Notation and Summations

FOR DISCUSSION: *Write down the closed form for each "summation formula:"*

$\sum_{k=1}^{n} 1 =$ $\qquad$ $\sum_{k=1}^{n} k =$ $\qquad$ $\sum_{k=1}^{n} k^2 =$ $\qquad$ $\sum_{k=1}^{n} k^3 =$

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1. Find the numerical value of each sum by writing out the terms and adding. (Do not use the "summation formulas" from the lesson.)

   (a) $\sum_{k=2}^{5} (2k-3) =$

   (b) $\sum_{k=-1}^{4} (k^2-2k) =$

2. Use the "summation formulas" to write each summation in closed form. Your answers should be in terms of n.

   (a) $\sum_{k=1}^{n} (4k^3-2) =$

   (b) $\sum_{k=1}^{n} (1+2k)^2 =$ $\qquad$ (**HINT**: First expand $(1 + 2k)^2$.)

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[1] This activity contains new material.
[†] This activity is referenced in Lesson 8.4.

3. Use the fomulas from your answers from Problem 2 to determine the following sums.

(a) $\sum_{k=1}^{63}(4k^3-2)=$

(b) $\sum_{k=1}^{40}(1+2k)^2=$

4. Express each summation in closed form in terms of n. Note that n is constant with respect to k, so it can be factored out of a summation if necessary.

(a) $\sum_{k=1}^{n}\left(2+\frac{1}{n}\cdot k\right)^2\left(\frac{1}{n}\right)=$ (**HINT**: Expand $\left(2+\frac{1}{n}\cdot k\right)^2\left(\frac{1}{n}\right)$.)

(b) $\sum_{k=1}^{n}3\left(-1+\frac{2}{n}\cdot k\right)^2\left(\frac{2}{n}\right)=$

5. Let F be a function, and suppose x_0, x_1, x_2, …, x_{n-1}, x_n are numbers in the domain of F. (Note that the subscripts are only labels for the numbers.) In Lesson 8.4, we will need to know a closed form for the "telescoping" sum

$$\sum_{k=1}^{n}\left(F(x_k)-F(x_{k-1})\right)$$

(a) Write out the first three terms and simplify.

$$\sum_{k=1}^{3}\left(F(x_k)-F(x_{k-1})\right)=$$

(b) Write out the first four terms and simplify.

$$\sum_{k=1}^{4}\left(F(x_k)-F(x_{k-1})\right)=$$

(c) Based on Parts (a) and (b), guess a closed form for $\sum_{k=1}^{n}\left(F(x_k)-F(x_{k-1})\right)$.

$$\sum_{k=1}^{n}\left(F(x_k)-F(x_{k-1})\right)=$$

6. (**OPTIONAL**) In mathematics, we cannot simply make a few correct guesses and conclude that our assumption will work in all cases. We must prove that our guess works for every n. Let's write the given sum as

$$\sum_{k=1}^{n}\left(F(x_k)-F(x_{k-1})\right)=\sum_{k=1}^{n}F(x_k)-\sum_{k=1}^{n}F(x_{k-1})$$

In the second sum on the righthand side, we could start the lower index one unit earlier, end the upper index one unit earlier, and replace the 'k' in the formula by k + 1. Therefore,

$$\sum_{k=1}^{n}\left(F(x_k)-F(x_{k-1})\right)=\sum_{k=1}^{n}F(x_k)-\sum_{k=0}^{n-1}F(x_k)$$

After "peeling off" the last term from the first sum and the first term from the last, we have

$$\sum_{k=1}^{n}\left(F(x_k)-F(x_{k-1})\right)=\left(F(x_n)+\sum_{k=1}^{n-1}F(x_k)\right)-\left(\sum_{k=1}^{n-1}F(x_k)+F(x_0)\right)=F(x_n)-F(x_0)$$

Does this answer match your guess from Part 3?