Activity 7.5 – Differential Equations

1. (a) $y = C_0 e^{5t}$

   (b) $y = C_0 e^{-3t}$

   (c) $y = C_1 e^{\sqrt{25}\,t} + C_2 e^{-\sqrt{25}\,t} = C_1 e^{5t} + C_2 e^{-5t}$

   (d) $y = C_1 \cos\left(\sqrt{15}\,t\right) + C_2 \sin\left(\sqrt{15}\,t\right)$

2. $T(t) = 70 + 115 e^{-kt}$;

   $T(5) = 70 + 115 e^{-k(5)} = 170$ implies $k = -\frac{1}{5}\ln\left(\frac{100}{115}\right)$;

   $T(15) = 70 + 115 e^{-\left(-\frac{1}{5}\ln\left(\frac{100}{115}\right)\right)(15)} \approx 146\,°F.$

3. $y = \frac{5}{4} e^{4t} + \frac{7}{4} e^{-4t}$

4. $y(t) = -1.5\cos\left(\frac{1}{2}t\right) + 10\sin\left(\frac{1}{2}t\right)$ in

5. $y = C_0 e^{\pm kt} \quad \rightarrow \quad y' = \pm k\left(C_0 e^{\pm kt}\right) = \pm ky$

6. $y' = \sqrt{k}\left(C_1 e^{\sqrt{k}\,t}\right) - \sqrt{k}\left(C_2 e^{-\sqrt{k}\,t}\right)$

   $y'' = k\left(C_1 e^{\sqrt{k}\,t}\right) + k\left(C_2 e^{-\sqrt{k}\,t}\right) = k\left(C_1 e^{\sqrt{k}\,t} + C_2 e^{-\sqrt{k}\,t}\right) = ky$

7. $y' = \sqrt{k}\left(-C_1 \sin\left(\sqrt{k}\,t\right)\right) + \sqrt{k}\left(C_2 \cos\left(\sqrt{k}\,t\right)\right)$

   $y'' = k\left(-C_1 \cos\left(\sqrt{k}\,t\right)\right) + k\left(-C_2 \sin\left(\sqrt{k}\,t\right)\right) = -k\left(C_1 \cos\left(\sqrt{k}\,t\right) + C_2 \sin\left(\sqrt{k}\,t\right)\right) = -ky$

8. (a) $\int \frac{1}{y}\,dy = \int \pm k\,dt \quad \rightarrow \quad \ln/y/ = \pm kt + C$

   (b) $\ln/y/ = \pm kt + C \quad \rightarrow \quad e^{\ln|y|} = |y| = e^{\pm kt + C} \quad \rightarrow \quad y = \pm e^{C} e^{\pm kt} \quad \rightarrow \quad C_0 = \pm e^{C}$