## Activity 7.5<sup>†‡</sup> – Differential Equations

**FOR DISCUSSION:** Which function has a first derivative proportional to itself? Which function has a second derivative proportional to itself? Which function has a second derivative proportional to its negative?

- 1. Write down the general solution to each differential equation.
  - (a)  $y' = 5y \longrightarrow y =$
  - (b)  $y' = -3y \longrightarrow y =$
  - (c)  $y'' = 25y \longrightarrow y =$
  - (d)  $y'' = -15y \rightarrow y =$
- 2. A cup of coffee at 185°F is allowed to cool in a room with a constant temperature of 70°F. If the temperature of the coffee is 170°F after 5 minutes, what will the temperature be after 15 minutes?

<sup>&</sup>lt;sup>†</sup> This activity is referenced in Lesson 7.5.

<sup>&</sup>lt;sup>‡</sup> This activity has supplemental exercises.

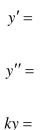
3. Find the general solution to the differential equation y'' - 16y = 0, and then find the constants  $C_1$  and  $C_2$  in the general solution given that y(0) = 3 and y'(0) = -2.

4. A mass attached to a vertical spring has position y(t) inches after t seconds, where y satisfies 4y" + y = 0. Positions below equilibrium and downward motion are considered positive. Find the position function if the initial position is 1.5 inches above equilibrium and the initial velocity is 5 in/s.

5. Suppose that  $y = 10e^{-7t}$ , and notice that  $y' = -7(10e^{-7t}) = -7y$ . This computation shows that  $y = 10e^{-7t}$  is a solution to the differential equation y' = -7y.

Perform a similar computation to verify that  $y = C_0 e^{\pm kt}$  is a solution to the differential equation  $y' = \pm ky$  for k > 0.

6. Verify that  $y = C_1 e^{\sqrt{k}t} + C_2 e^{-\sqrt{k}t}$  is a solution to the differential equation y'' = ky for k > 0.



- 7. Verify that  $y = C_1 \cos(\sqrt{k} t) + C_2 \sin(\sqrt{k} t)$  is a solution to the equation y'' = -ky for k > 0.
  - y' =y'' =-ky =
- 8. (OPTIONAL) In Part 5, we merely verified that any function of the form  $y = C_0 e^{\pm kt}$  is a solution to the differential equation  $y' = \pm ky$ . We will now prove that these are the *only* solutions. First we write  $y' = \pm ky$  in Leibniz notation as  $\frac{dy}{dt} = \pm ky$ . Then we "separate" the *t*'s and *y*'s to get

$$\frac{1}{y}dy = \pm kdt$$

(a) Integrate the left-hand side with respect to *y*, and integrate the right-hand side with respect to *t*. This will introduce a logarithm on the left-hand side. You need only include one constant of integration *C*, so place it on the right-hand side.

(b) Solve the result from Part (a) for y. Try to identify  $C_0$ .