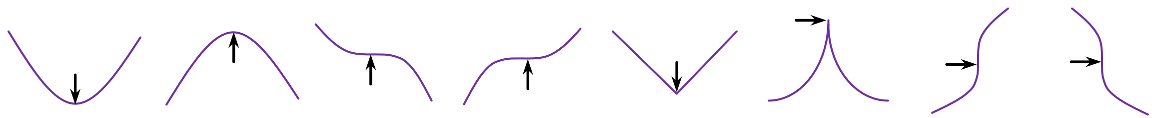
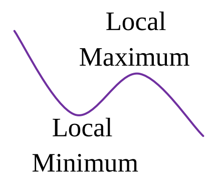
Lesson 7.2 – Graph Analysis Using First and Second Derivatives

A point *in the domain* of f at which $f'(x)=0$ (horizontal tangent) or $f'(x)$ is undefined (vertical tangent or no tangent) is called a **critical point** of f. Here are some examples:



Note that a graph may have a peak, valley, saddle point, inflection point, a change in increase and decrease, or a change in concavity at a critical point.

Suppose x_0 is a point in the domain of f. If $f(x_0) \geq f(x)$ for all x in some open interval containing x_0, then f has a **local (or relative) maximum** at x_0. If $f(x_0) \leq f(x)$ for all x in some open interval containing x_0, then f has a **local (or relative) minimum** at x_0. Local maxima and minima are collectively called **local extrema**.



Using the first derivative for graph analysis: $f'(x)$ is the slope (rate of change) of f at x.

(a) $f'(x)>0$ on an interval I $\rightarrow$ f is increasing on I
(b) $f'(x)<0$ on an interval I $\rightarrow$ f is decreasing on I

First derivative test: Let x_0 be a critical point of f.

(a) $f'(x)$ changes sign from $+$ to $-$ at x_0 $\rightarrow$ f has a local maximum at x_0
(b) $f'(x)$ changes sign from $-$ to $+$ at x_0 $\rightarrow$ f has a local minimum at x_0
(c) $f'(x)$ does not change sign at x_0 $\rightarrow$ f does not have a local extremum at x_0

Using the second derivative for graph analysis: $f''(x)$ is the slope (rate of change) of f' at x.

(a) $f''(x)>0$ on an interval I $\rightarrow$ f' is increasing on I $\rightarrow$ f is concave up on I
(b) $f''(x)<0$ on an interval I $\rightarrow$ f' is decreasing on I $\rightarrow$ f is concave down on I
(c) $f''(x)$ changes sign at domain point x_0 $\rightarrow$ f has an inflection point at x_0

Second derivative test: Suppose f is twice differentiable at critical point x_0 and $f'(x_0)=0$.

(a) $f''(x_0)>0$ $\rightarrow$ f has a local minimum at x_0
(b) $f''(x_0)<0$ $\rightarrow$ f has a local maximum at x_0
(c) $f''(x_0)=0$ $\rightarrow$ no conclusion (f may or may not have a local extremum at x_0)