Activity 7.2‡ – Graph Analysis Using First and Second Derivatives

FOR DISCUSSION: *What is a critical point of a function?*
What does f' tell us about the graph of f?
What does f'' tell us about the graph of f?

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1. Let $f(x) = (10 - 2x)e^x$.

   (a) Compute f'. Use sign tests to help determine the following.
   - (i) Critical numbers
   - (ii) Local maxima
   - (iii) Local minima
   - (iv) Interval(s) of increase
   - (v) Interval(s) of decrease

   (b) Compute f''. Use sign tests to help determine the following.
   - (i) Inflection points
   - (ii) Interval(s) of upward concavity
   - (iii) Interval(s) of downward concavity

‡ This activity has supplemental exercises.

(c) Use the formula for f to help determine the following.

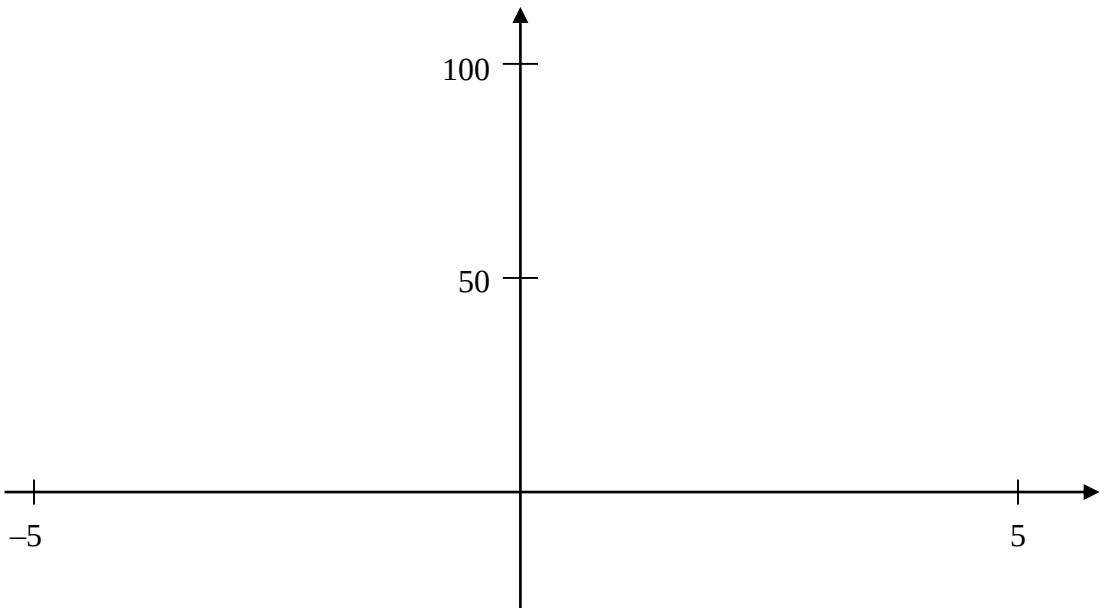
(i) x-intercept(s) (set $f(x) = 0$)

(ii) y-intercept (let $x = 0$)

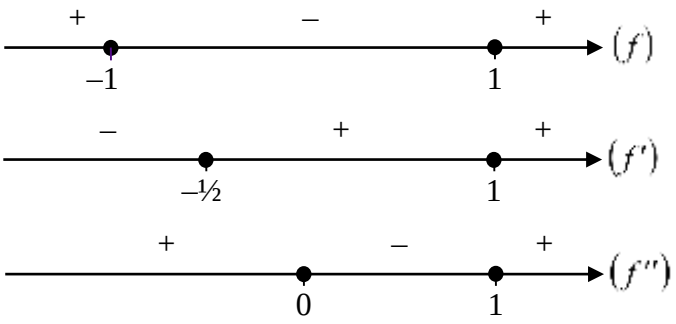
(iii) End behavior (limit at infinity and limit at negative infinity)

(Hint: For the limit at $-\infty$, you should write f as $f(x) = \dfrac{10 - 2x}{e^{-x}}$.)

(d) Use the information you found in this problem to sketch a graph of f below. Note the units on the axes. (**Hint**: Find the y-coordinates for the extrema and inflection points and plot these points.)

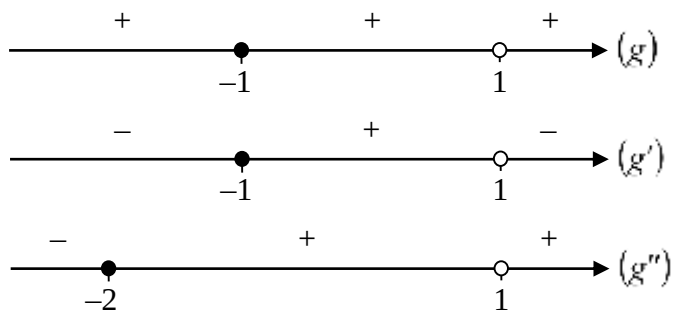


2. Sign charts for a function f and its first two derivatives are given. A black dot represents a zero. Assume that the graph of f has no vertical or horizontal asymptotes, and that the graph has a y-intercept at the point $(0, -\frac{1}{2})$. Sketch a possible graph of f.



–1	0	1	

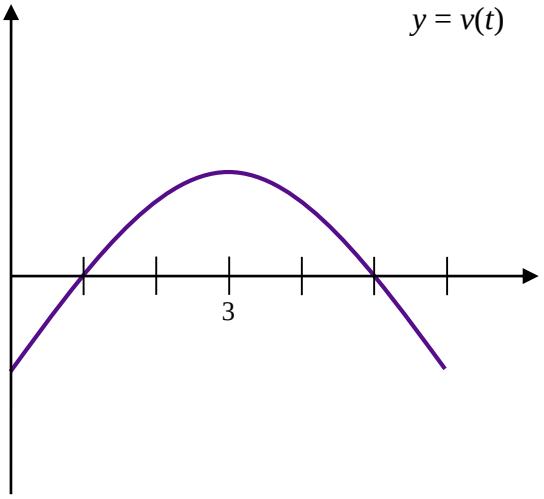
3. Sign charts for a function g and its first two derivatives are given. A black dot represents a zero, and a white dot represents a vertical asymptote. Assume that the graph of g has a vertical asymptote at $x = 1$, a horizontal asymptote in both directions at $y = 2$, and a y-intercept at $(0, \frac{1}{2})$. Sketch a possible graph of g.

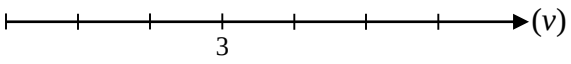


–2	–1	0	1	2	

4. The graph of the *velocity* $v(t)$ of an object is given on the interval [0, 6]. Assume that the object is at the "origin" at $t = 0$ and $t = 2$. Fill in the sign charts, and use this information to sketch possible graphs of acceleration $a(t)$ and position $s(t)$. Then fill in the blanks.

(a) $y = v(t)$



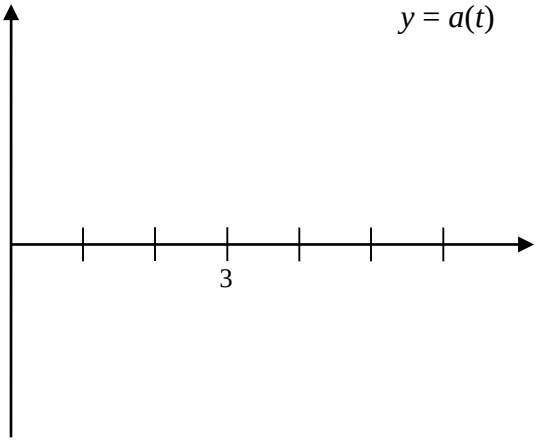


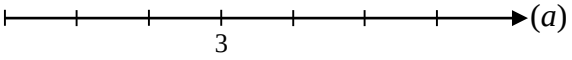
Moving to left: _____

Moving to right: _____

At rest: _____

(b) $y = a(t)$



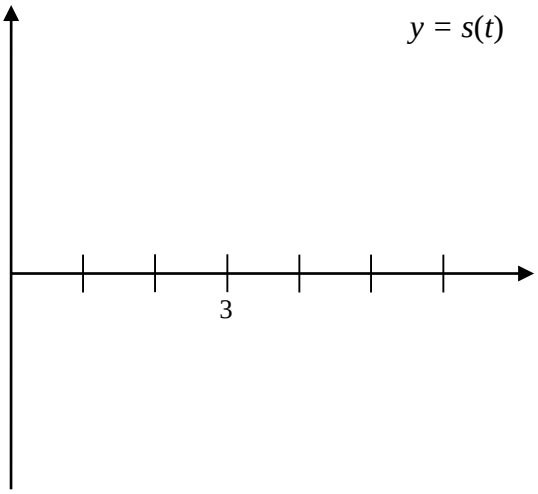


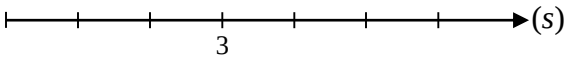
Speeding up: _____

Slowing down: _____

(Careful here! You cannot merely look at the signs of acceleration. You must also look at the signs of velocity.)

(c) $y = s(t)$





Left of origin: _____

Right of origin: _____

At origin: _____