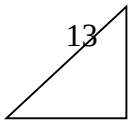
Activity 7.1 – Related Rates

1. **Related Variables Equation**: $x^2 + y^2 = 13^2$



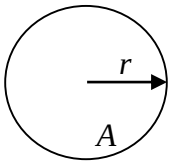
y

Given: $\frac{dx}{dt} = 1$; **Want**: $\frac{dy}{dt}$ when $x = 12$ and $y = \sqrt{169 - x^2} = \sqrt{169 - 144} = 5$

Related Rates Equation: $2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0 \rightarrow 2(12)(1) + 2(5)\frac{dy}{dt} = 0 \rightarrow \frac{dy}{dt} = -2.4$

Answer: When the bottom of the ladder is 12 feet from the wall, the top of the ladder is falling at a rate of 2.4 ft/s.

2. **Related Variables Equation**: $A = \pi r^2$

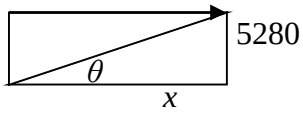


Given: $\frac{dA}{dt} = 4$; **Want**: $\frac{dr}{dt}$ when $A = 10$ and $r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{10}{\pi}}$

Related Rates Equation: $\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt} \rightarrow 4 = 2\pi\sqrt{\frac{10}{\pi}} \cdot \frac{dr}{dt} \rightarrow \frac{dr}{dt} \approx 0.36$

Answer: When the area 10 mi^2, the radius of the spill is increasing at a rate of 0.36 mi/h.

3. **Related Variables Equation**: $\cot\theta = \frac{x}{5280}$

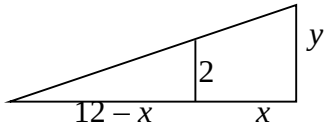


Given: $\frac{dx}{dt} = 400$; **Want**: $\frac{d\theta}{dt}$ when $\theta = 30° = \frac{\pi}{6}$

Related Rates Equation: $-\csc^2\theta \cdot \frac{d\theta}{dt} = \frac{1}{5280} \cdot \frac{dx}{dt} \rightarrow -\frac{1}{\sin^2\left(\frac{\pi}{6}\right)} \cdot \frac{d\theta}{dt} = \frac{1}{5280} \cdot 400 \rightarrow \frac{d\theta}{dt} \approx -0.02$

Answer: When the angle of elevation θ is 30°, θ is decreasing at a rate of 0.02 rad/s.

4. **Related Variables Equation**: $\frac{12-x}{2} = \frac{12}{y}$ or $6 - \frac{x}{2} = \frac{12}{y}$

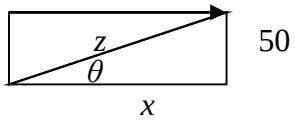


Given: $\frac{dx}{dt} = -1.2$; **Want**: $\frac{dy}{dt}$ when $x = 6$ and $y = \frac{24}{12-6} = 4$

Related Rates Equation: $-\frac{1}{2} \cdot \frac{dx}{dt} = -\frac{12}{y^2} \cdot \frac{dy}{dt} \rightarrow -\frac{1}{2} \cdot (-1.2) = -\frac{12}{(4)^2} \cdot \frac{dy}{dt} \rightarrow \frac{dy}{dt} = -0.8$

Answer: When the woman is 6 meters from the building, her shadow is decreasing at a rate of –0.8 m/s.

5. **Related Variables Equation**: $x^2 + 50^2 = z^2$



Given: $\frac{dx}{dt} = 2$; **Want**: $\frac{dz}{dt}$ when $\theta = \frac{\pi}{6}$ and $\frac{x}{z} = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Related Rates Equation: $2x \cdot \frac{dx}{dt} = 2z \cdot \frac{dz}{dt} \rightarrow \frac{dz}{dt} = \frac{x}{z} \cdot \frac{dx}{dt}\ \frac{\sqrt{3}}{2} \cdot 2 = \sqrt{3} \approx 1.73$

Answer: When the angle of elevation is $\pi/6$ radians, the string must be let out at a rate of 1.73 ft/s.

6. **Related Variables Equation**: $P \cdot V^{1.2} = C$

Given: $\frac{dP}{dt} = -10$; **Want**: $\frac{dV}{dt}$ when $V = 330$ and $P = 90$

RRE: $\frac{dP}{dt} \cdot V^{1.2} + P \cdot \left(1.2V^{0.2}\right) \cdot \frac{dV}{dt} = 0 \rightarrow (-10) \cdot (330)^{1.2} + (90) \cdot \left(1.2(330)^{0.2}\right) \cdot \frac{dV}{dt} = 0$

Answer: At this instant, the volume is increasing by 30.56 cm^3/min.