Chapter 7 Review

1. **(Lesson 7.1)** A 20-ft ladder is leaning against a wall, and the top of the ladder is sliding down the wall at a constant rate of 1 ft/s. How fast is the bottom of the ladder sliding away from the wall when the top of the ladder is 8 ft above the ground?

2. **(Lesson 7.1)** Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of 8 m/s, then how fast is the area of the spill increasing when the radius is 200 m?

3. **(Lesson 7.2)** Suppose that $f(x) = 3x^4 - 16x^3 - 192x^2 + 49$. Practice showing all work, number lines, and sign tests. Complete this problem without a graphing calculator.

   (a) Find all critical points of *f*.

   (b) Use interval notation to indicate where *f* is increasing.

   (c) Use interval notation to indicate where *f* is decreasing.

   (d) Find the *x*-values of all local maxima of *f*.

   (e) Find the *x*-values of all local minima of *f*.

   (f) Use interval notation to indicate where *f* is concave up.

   (g) Use interval notation to indicate where *f* is concave down.

   (h) Find the *x*-values of all inflection points of *f*.

   (i) Use all of the preceding information to sketch a graph of *f*.

4. **(Lesson 7.4)** Let $f(x) = 2x^3 - 15x^2 - 216x$ on the interval [–5, 16]. Complete this problem without a graphing calculator.

   (a) The derivative of *f* is $f'(x) =$ _____.

   (b) The critical points of *f* are *x* = _____.

   (c) The *y*-values corresponding to the critical points and endpoints are *y* = _____.

   (d) According to the Extreme Value Theorem, the minimum value of *f* on [–5, 16] is *y* = _____, the minimum value occurs at *x* = _____, and this *x* is a(n) _____ (Critical point? Endpoint?).

   (e) According to the Extreme Value Theorem, the maximum value of *f* on [–5, 16] is *y* = _____, the maximum value occurs at *x* = _____, and this *x* is a(n) _____ (Critical point? Endpoint?).

5. **(Lesson 7.4)** An open-top box is to be constructed with 1500 cm^2 of material. The length of the base is to be twice the width. Find the largest possible volume of the box.

6. **(Lesson 7.4)** A rancher wants to fence in an area of 2,000,000 square feet in a rectangular field and then divide it in half with a fence down the middle parallel to one side. What is the shortest length of fence that the rancher can use?

7. **(Lesson 7.4)** A cylindrical can without a top must be constructed to contain $V = 2000$ cm^3 of liquid. Find the base radius r and height h that will minimize the amount of material needed to construct the can.

8. **(Lesson 7.5)** Find y as a function of t if $16y'' - 9y = 0$, $y(0) = 1$, and $y'(0) = 4$. (HINT: First solve $16y'' - 9y = 0$ for y''.)

9. **(Lesson 7.5)** Suppose an initial population of 1500 honeybees is changing exponentially at a rate of –4.5% per day. That is, suppose $\frac{dP}{dt} = -0.045P$, where $P(0) = 1500$. Find a formula for the population after t days.

10. **(Lesson 7.5)** A weight is attached to a horizontal spring that satisfies the differential equation $x'' = -0.01x$. The units for x are centimeters, and the units for the independent variable t are seconds. Initially the spring is stretching at a rate of 4 cm/s and is compressed 3 cm from equilibrium. Assume that compression is negative and stretching is positive.

    (a) Write a formula for the location of the weight at time t.

    (b) Find the location of the weight 5 seconds after it is set in motion. Is the spring stretched or compressed at this time?