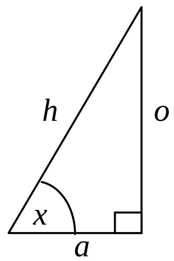
Lesson 6.3 – Other Trigonometric Functions

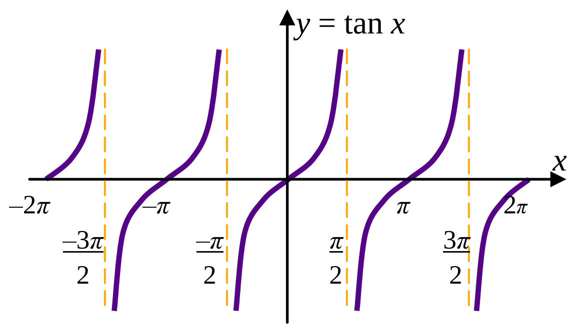
Right triangle trigonometry: The six trigonometric functions—cosine, sine, tangent, secant, cosecant, and cotangent—are defined in terms of ratios of the lengths of the sides of an acute right triangle:



$$\cos x = \frac{a}{h} \qquad \sin x = \frac{o}{h} \qquad \tan x = \frac{o}{a} = \frac{\sin x}{\cos x}$$

$$\sec x = \frac{h}{a} = \frac{1}{\cos x} \qquad \csc x = \frac{h}{o} = \frac{1}{\sin x} \qquad \cot x = \frac{a}{o} = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

The **tangent function** is an important function:



Domain: The set of all real numbers except the odd integer multiples of $\pi/2$. The graph has vertical asymptotes at these points.

Range: The set of all real numbers.

Roots: $\tan x = 0$ at every integer multiple of π.

Graph: Continuous on its domain; vertical asymptotes at odd integer multiples of $\pi/2$.

Periodicity and parity: Tangent is periodic with period π, and tangent is **odd**, since

$$\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x} = -\tan x$$

Derivatives of the six trigonometric functions and corresponding antiderivatives:
In Activity 6.3, you will use the quotient or chain rules to derive some of the differentiation formulas. Of the six trigonometric functions, we only have antiderivatives of sine and cosine.

$$\frac{d}{dx}(\sin x) = \cos x \qquad \int \cos x\, dx = \sin x + C$$

$$\frac{d}{dx}(\cos x) = -\sin x \qquad \int \sin x\, dx = -\cos x + C$$

$$\frac{d}{dx}(\tan x) = \frac{1}{\cos^2 x} = \sec^2 x \qquad \int \sec^2 x\, dx = \tan x + C$$

$$\frac{d}{dx}(\cot x) = -\frac{1}{\sin^2 x} = -\csc^2 x \qquad \int \csc^2 x\, dx = -\cot x + C$$

$$\frac{d}{dx}(\sec x) = \frac{\sin x}{\cos^2 x} = \sec x \tan x \qquad \int \sec x \tan x\, dx = \sec x + C$$

$$\frac{d}{dx}(\csc x) = -\frac{\cos x}{\sin^2 x} = -\csc x \cot x \qquad \int \csc x \cot x\, dx = -\csc x + C$$

Note: To find the derivative of a co-function, "co- and negate" the derivative of the original.