Lesson 6.2 – Derivatives and Antiderivatives of Cosine and Sine

Degrees and radians: Angles are typically measured in terms of **degrees** or **radians**:

1 degree = A plane angle representing $\frac{1}{360}$ of a full rotation around the circle.

1 radian = A plane angle representing a length of arc equal to the radius of the circle.

Conversions: Once around the circle is 360° or 2π radians. Thus, 1 radian $= \frac{180^\circ}{\pi} \approx 57.3^\circ$, and

D degrees to radians: $D^\circ \cdot \dfrac{\pi}{180^\circ}$ R radians to degrees: $R \cdot \dfrac{180^\circ}{\pi}$

Important note: When differentiating or integrating any trigonometric function, the argument x must be in radians, not degrees. Be sure your calculator is set to radian mode!

In order to obtain the derivative formula for cosine, we will need three basic facts:

1. Graphically, we can see that the slope of the cosine graph at $x = 0$ is 0:

$$\lim_{\Delta x \to 0} \frac{\cos(0+\Delta x)-\cos(0)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cos(\Delta x)-1}{\Delta x} = 0$$

2. According to the TI-84, the slope of the sine graph at $x = 0$ is 1:

$$\lim_{\Delta x \to 0} \frac{\sin(0+\Delta x)-\sin(0)}{\Delta x} = \lim_{\Delta x \to 0} \frac{\sin(\Delta x)}{\Delta x} = 1$$

3. A sum identity: $\cos(x+\Delta x) = \cos x \cdot \cos(\Delta x) - \sin x \cdot \sin(\Delta x)$

By the definition of the derivative and the facts above,

$$\begin{aligned}
\frac{d}{dx}(\cos x) &= \lim_{\Delta x \to 0} \frac{\cos(x+\Delta x)-\cos x}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\cos x \cdot \cos(\Delta x) - \sin x \cdot \sin(\Delta x) - \cos x}{\Delta x} \\
&= \lim_{\Delta x \to 0} \frac{\cos x \cdot (\cos(\Delta x)-1)}{\Delta x} - \lim_{\Delta x \to 0} \frac{\sin x \cdot \sin(\Delta x)}{\Delta x} \\
&= \cos x \cdot \lim_{\Delta x \to 0} \frac{\cos(\Delta x)-1}{\Delta x} - \sin x \cdot \lim_{\Delta x \to 0} \frac{\sin(\Delta x)}{\Delta x} \\
&= -\sin x
\end{aligned}$$

Also, $\frac{d}{dx}(\sin x) = \frac{d}{dx}\left(\cos\left(\frac{\pi}{2} - x\right)\right) = -\sin\left(\frac{\pi}{2} - x\right) \cdot (-1) = \sin\left(\frac{\pi}{2} - x\right) = \cos x$, by the cofunction identities.

Derivatives and antiderivatives of cos(x) and sin(x):

$$\frac{d}{dx}(\sin(x)) = \cos(x) \qquad \int \cos(x)\,dx = \sin(x) + C$$

$$\frac{d}{dx}(\cos(x)) = -\sin(x) \qquad \int \sin(x)\,dx = -\cos(x) + C$$