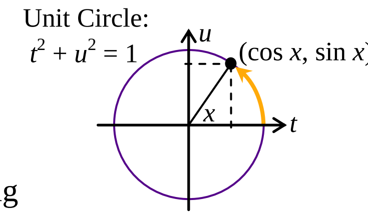
Lesson 6.1 – The Cosine and Sine Functions

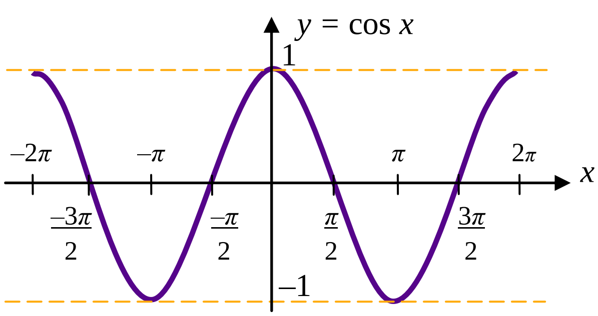


Functions that model a vibrating spring, an electrical current, and the horizontal range of a kicked soccer ball involve the two most important trigonometric functions. In the unit circle, $t^2 + u^2 = 1$, a radius lying along the positive t-axis creates an angle x by sweeping counterclockwise around the circle. The first coordinate of the point on the circle is the **cosine** of x, and the second coordinate is the **sine** of x. Since $(\cos x, \sin x)$ is a point on the unit circle, the coordinates satisfy the equation of the circle, which yields the

Pythagorean identity: $\cos^2 x + \sin^2 x = 1$ (**Note:** $\text{trig}^2 x$ is shorthand for $(\text{trig}\, x)^2$)

Once around the unit circle measures 2π units, where $\pi \approx 3.141593$.

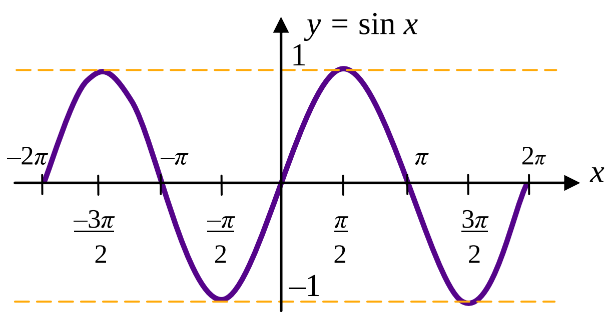
Cosine function: $y = \cos x$
Domain: The set of all real numbers.
Range: $-1 \le \cos x \le 1$ for all x.
Roots: $\cos x = 0$ at odd integer multiples of $\pi/2$.
Graph: Continuous everywhere; period 2π.



Parity: Cosine is **even**; i.e., $\cos(-x) = \cos x$.
Cofunction identity: The cosine graph is a shift of sine by $\pi/2$ units to the left:

$$\cos x = \sin\left(x + \tfrac{\pi}{2}\right) = \sin\left(\tfrac{\pi}{2} - x\right)$$

Sine function: $y = \sin x$
Domain: The set of all real numbers.
Range: $-1 \le \sin x \le 1$ for all x.
Roots: $\sin x = 0$ at integer multiples of π.
Graph: Continuous everywhere; period 2π.



Parity: Sine is **odd**; i.e., $\sin(-x) = -\sin x$.
Cofunction identity: The sine graph is a shift of cosine by $\pi/2$ units to the right:

$$\sin x = \cos\left(x - \tfrac{\pi}{2}\right) = \cos\left(\tfrac{\pi}{2} - x\right)$$

General forms: $y = A\cos(Bx - C) = A\cos\left(B\left(x - \tfrac{C}{B}\right)\right)$ and $y = A\sin(Bx - C) = A\sin\left(B\left(x - \tfrac{C}{B}\right)\right)$.

Transformations: The general forms are obtained from $y = \cos x$ and $y = \sin x$ as follows:

- $|A|$ is a vertical stretch or compression called the **amplitude**. $A < 0$ implies x-axis reflection.
- $|B|$ is a horizontal stretch or compression. If $B < 0$, then its sign can be changed using parity.
- $2\pi/|B|$ is the **period**, which tells the smallest interval after which the graph repeats.
- $|B|/2\pi$ is called the **frequency**. Note that frequency = 1/period.
- C/B is a horizontal shift sometimes called the **phase** or **phase shift**, but this terminology may refer only to C in certain contexts.