Examples 6.1 – The Cosine and Sine Functions

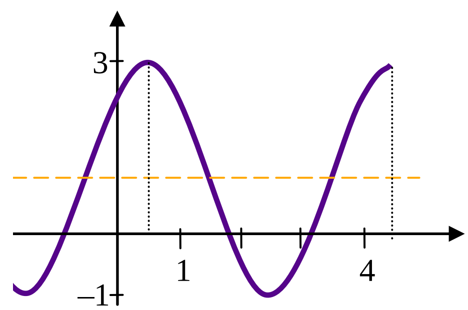
1. Suppose $y=-4\sin\left(3x+\frac{7\pi}{5}\right)=-4\sin\left(3\left(x+\frac{7\pi}{15}\right)\right)$. Determine the amplitude, period, and phase shift of the graph.

   Solution: Then the amplitude is 4, the period is $\frac{2\pi}{3}$, and the phase shift is $-\frac{7\pi}{15}$, which is a shift to the left of $\frac{7\pi}{15}$ units.

2. Let us call the horizontal line about which the graph oscillates the **midline**. With no vertical shift, the midline of a general cosine or sine function is the line $y = 0$. Determine the amplitude, period, phase shift, and midline of the sinusoidal function $y=5\cos(3\pi x)-2$.

   Solution: The amplitude is 5, and the period is $\frac{2\pi}{3\pi}=\frac{2}{3}$. The graph has no phase shift, but it does have a vertical shift of 2 units downward, so the midline is $y = -2$.

3. For the graph shown on the right, find a sinusoidal formula of the form $y=A\cos(Bx-C)+D$.



   Solution: By inspection, the midline is at $D = 1$, the amplitude is $A = 2$, and the period is $\frac{2\pi}{B}=4$ (so $B = \frac{\pi}{2}$). The graph has been shifted to the right by $\frac{1}{2}$ unit, which means that $\frac{C}{B}=\frac{1}{2}$. Therefore, $C = \frac{\pi}{4}$ and the equation is

$$y=2\cos\left(\frac{\pi}{2}x-\frac{\pi}{4}\right)+1=2\cos\left(\frac{\pi}{2}\left(x-\frac{1}{2}\right)\right)+1$$

4. We know that $y=\sin x$ is zero for $x = n\pi$ (n an integer), and $y=\cos x$ is zero for $x=\frac{m\pi}{2}$ (m an odd integer). What are the roots (x-intercepts) of $y=\sin\left(\frac{3\pi}{2}x-1\right)$ and $y=\cos\left(5x+\frac{\pi}{2}\right)$?

   Solution: To solve $\sin\left(\frac{3\pi}{2}x-1\right)=0$, we set the argument $\frac{3\pi}{2}x-1=n\pi$ and solve for x:

$$\frac{3\pi}{2}x-1=n\pi$$
$$\frac{3\pi}{2}x=n\pi+1$$
$$x=\frac{2}{3\pi}(n\pi+1), \text{ where } n \text{ is an integer.}$$

   To solve $\cos\left(5x+\frac{\pi}{2}\right)=0$, we set $5x+\frac{\pi}{2}=\frac{m\pi}{2}$ and solve for x. Here, we have

$$5x+\frac{\pi}{2}=\frac{m\pi}{2}$$
$$5x=\frac{m\pi}{2}-\frac{\pi}{2}$$
$$x=\frac{\pi}{10}(m-1), \text{ where } m \text{ is an odd integer.}$$