Activity 6.1[1†] – The Cosine and Sine Functions

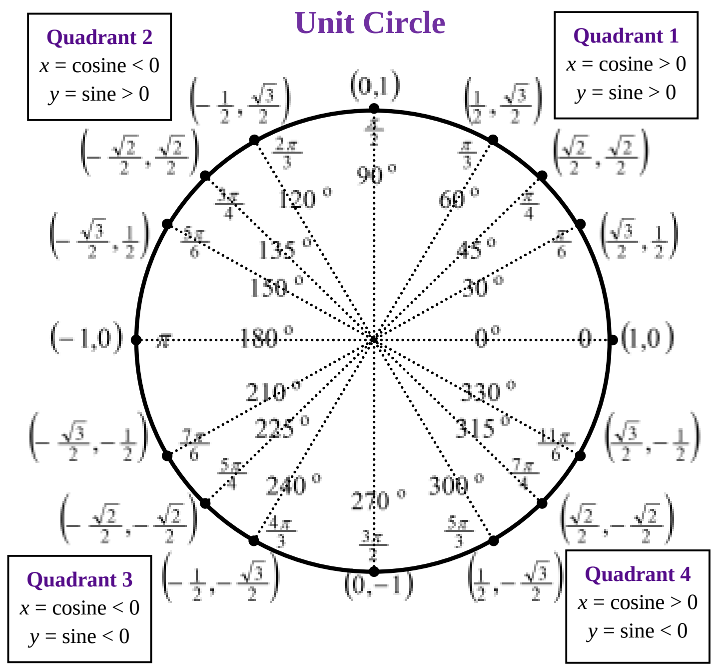
FOR DISCUSSION: *Give a geometrical interpretation of the cosine and sine functions.*
In words, describe the following aspects of a sinusoidal graph:
Amplitude, period, frequency, phase shift, midline.

In Lesson 6.1 we learned that a radius lying along the positive horizontal axis creates a positive angle θ by sweeping counterclockwise around the unit circle. The first coordinate of the point on the circle is the **cosine** of θ, and the second coordinate is the **sine** of θ. When θ is a multiple of $\frac{\pi}{6}(=30^\circ)$ or $\frac{\pi}{4}(=45^\circ)$, we refer to it as a **basic angle**. The cosines and sines of the basic angles in the first quadrant are easy to remember since they each have the form $\frac{\sqrt{n}}{2}$, where n is an integer ranging from 0 through 4. The remaining basic angles can be found by symmetry with the angles in the first quadrant. For instance, $\cos\left(\frac{4\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$ since the corresponding points have opposite x-coordinates, and $\sin\left(\frac{5\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right)$ since the corresponding points have the same y-coordinates. The table and diagram below summarize the basics.

Basic angles (in first quadrant)

θ	$x = \cos\theta$	$y = \sin\theta$
$0 = 0^\circ$	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sqrt{0}}{2} = 0$
$\frac{\pi}{6} = 30^\circ$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$
$\frac{\pi}{4} = 45^\circ$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3} = 60^\circ$	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2} = 90^\circ$	$\frac{\sqrt{0}}{2} = 0$	$\frac{\sqrt{4}}{2} = 1$

Unit Circle



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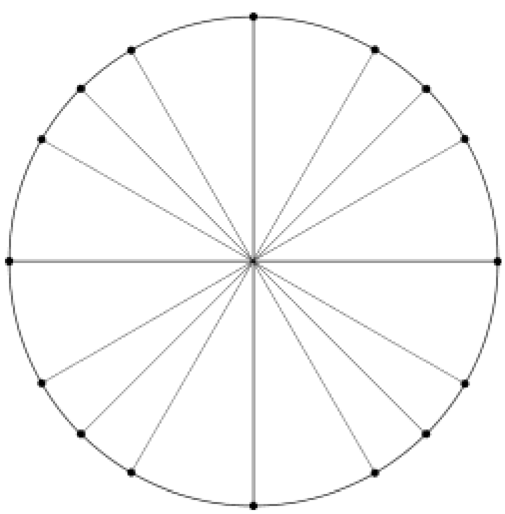
[1] This activity contains new content.
[†] This activity is referenced in Activity 6.2.

In Lesson 6.3, we will see that the **tangent** function is defined as $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. Therefore, finding the tangent of an angle θ is simply a matter of dividing its sine by its cosine.

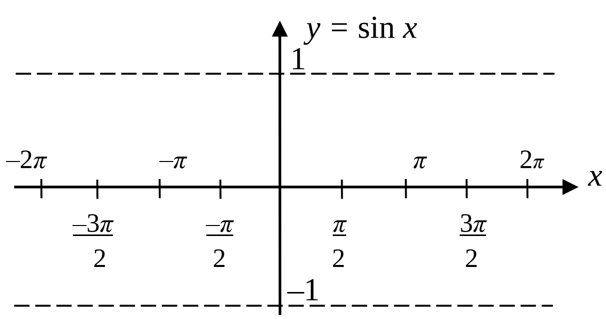
1. Without looking back, complete the table of basic angles, and use it to fill in the given table. Use the unit circle below if necessary.

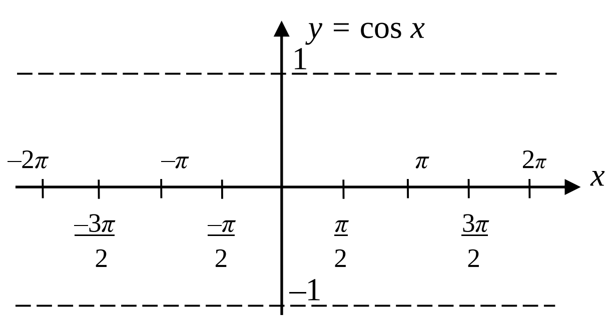
θ	$x = \cos\theta$	$y = \sin\theta$
$0 = 0^\circ$	$\frac{\sqrt{4}}{2} = 1$	$\frac{\sqrt{0}}{2} = 0$
$\frac{\pi}{6} = 30^\circ$		
$\frac{\pi}{4} = 45^\circ$		
$\frac{\pi}{3} = 60^\circ$		
$\frac{\pi}{2} = 90^\circ$		

θ	$\cos\theta$	$\sin\theta$	$\tan\theta$
$0 = 0^\circ$			
$\frac{\pi}{6} = 30^\circ$			
$\frac{\pi}{4} = 45^\circ$			
$\frac{\pi}{3} = 60^\circ$			
$\frac{\pi}{2} = 90^\circ$			
$\pi = 180^\circ$			
$\frac{3\pi}{2} = 270^\circ$			
$\frac{5\pi}{6} = 150^\circ$			
$\frac{4\pi}{3} = 240^\circ$			



2. Sketch the graphs of the sine and cosine functions below. Use the table above if necessary.





3. Suppose $y = 4\cos(3x+2)-1$. Determine each of the following.

(a) Amplitutde

(b) Period

(c) Phase (horizontal shift)

(d) Midline (vertical shift)

(e) Sketch the graph of y below.

4. Suppose that you were sitting on a pier at your favorite ocean resort on July 22 at 9:00 a.m., just as high tide was occurring. A nearby tide gauge showed the water level at 3.96 meters. Assuming that the interval between high and low tides at this resort is 5.5 hours, you decided to return to the pier at 2:30 p.m. on that same day to check the water level and observed it to be 1.60 meters. (**HINT**: Make a sketch!)

   (a) Write an equation of the form $h(t) = A\cos(Bt) + D$ that models the height of the tide at the resort t hours after 9:00 a.m. on July 22 (i.e., $t = 0$ corresponds to 9:00 a.m.). (**HINT**: Use the given information to determine A, B, and D.)

   (b) Use your model to find the height of the tide at 3:00 a.m. on July 23, and at 8:00 a.m. on July 24.

   (c) Write an equation of the form $H(t) = A\cos(Bt - C) + D$ that models the height of the tide at the resort t hours after *noon* on July 22 (i.e., $t = 0$ corresponds to 12:00 p.m.). (**HINT**: Use the fact that the new model is the same as the one in Part (a) except for an appropriate phase shift.)