Activity 5.6[‡] – Definite Integrals of Exponentials and Logarithms

FOR DISCUSSION: *Describe the left-hand, right-hand, and midpoint approximations.*
When must we use a numerical approximation for a definite integral?
How does the Fundamental Theorem help us avoid approximations?

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1. Suppose we want to use a midpoint approximation and $n = 5$ subintervals to estimate the integral $\int_0^2 3e^{x^2}\,dx$. (There exists no formula for the antiderivative of e^{x^2}, so we have no choice but to approximate it!)

   (a) The width of each subinterval is $\Delta x =$ __________ .

   (b) The midpoint approximation is

   $$\int_0^2 3e^{x^2}\,dx \approx (\_\_\_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_\_\_ + \_\_\_\_\_\_\_\_\_\_)\cdot \_\_\_\_\_\_\_$$

   $=$ __________________ (to 6 decimal places)

2. Use the Fundamental Theorem to evaluate each integral.

   (a) $\int_{-1}^{1} e^{5.2x}\,dx =$

   (b) $\int_0^1 10^x\,dx =$

[‡] This activity has supplemental exercises.

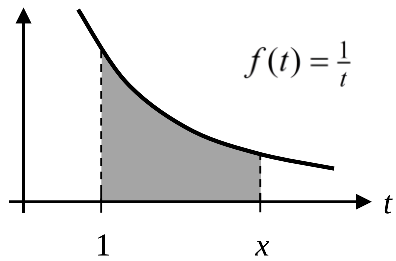
3. An oil tanker ruptured and started leaking at an exponential rate of $R(t) = ab^t$ million gallons per minute, where a is the initial rate, and t is minutes after the rupture. At the moment the tank ruptured, oil was leaking out at a rate of 0.5 million gallons per minute, and R decayed with a half-life of 120 minutes (i.e., $R(120) = 0.25$).

   (a) Use the given information to find a formula for $R(t)$, with units.

   (b) At what rate was the oil leaking after 280 minutes?

   (c) How much oil leaked out over the first 280 minutes? (Think accumulated change…)

4. (OPTIONAL) Consider the graph of $f(t) = \frac{1}{t}$, shown on the right. Suppose we want to find the area beneath the graph from 1 to $x > 1$.



(a) Set up a definite integral with respect to t that represents this area. Evaluate the integral.

(b) In Lesson 5.4, we defined the logarithm as the inverse of the exponential function. We could have just as well defined the function $y = \ln x$ as

$$\ln x = \int_1^x \frac{1}{t}\,dt,$$

and then defined the natural exponential as the inverse of the natural logarithm. But why base e? You may be wondering where Euler's number fits into all this… Find x such that the area under $y = \frac{1}{t}$ from $[1, x]$ is exactly equal to 1. That is, solve the equation $\int_1^x \frac{1}{t}\,dt = 1$ for x.

NOTE: Rather than define e as $\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^n$, we could have defined $\ln x$ as $\int_1^x \frac{1}{t}\,dt$ and then defined e as the unique number such that $\ln x = 1$.