



Examples 5.5 – Derivatives and Antiderivatives of Exponentials and Logarithms

1. Compute each of the following derivatives. Assume y is a function of x .

$$(a) \frac{d}{dx}(\ln y) \qquad (b) \frac{d}{dx}(\ln(x^2 + 3x - 2)) \qquad (c) \frac{d}{dx}(\log_{10}(5x - 1))$$

Solution: (a) $\frac{d}{dx}(\ln y) = \frac{1}{y} \cdot y' = \frac{y'}{y}$

$$(b) \frac{d}{dx}(\ln(x^2 + 3x - 2)) = \frac{2x + 3}{x^2 + 3x - 2}$$

$$(c) \frac{d}{dx}(\log_{10}(5x - 1)) = \frac{1}{\ln 10} \cdot \frac{5}{5x - 1} = \frac{5}{(\ln 10)(5x - 1)}$$

2. Compute each of the following derivatives. Assume y is a function of x .

$$(a) \frac{d}{dx}(e^y) \qquad (b) \frac{d}{dx}(b^y) \qquad (c) \frac{d}{dx}(10^{x^2})$$

Solution: (a) $\frac{d}{dx}(e^y) = e^y \cdot y'$

$$(b) \frac{d}{dx}(b^y) = b^y \cdot \ln b \cdot y'$$

$$(c) \frac{d}{dx}(10^{x^2}) = 10^{x^2} \cdot \ln 10 \cdot 2x = (\ln 100)x \cdot 10^{x^2}$$

3. Compute each of the following antiderivatives.

$$(a) \int 10^x dx \qquad (b) \int \frac{1.622}{x} dx \qquad (c) \int \frac{3x^3 - 5x^2 + 2x - 4}{x^2} dx$$

Solution: (a) $\int 10^x dx = \frac{10^x}{\ln 10} + C$

$$(b) \int \frac{1.622}{x} dx = 1.622 \int \frac{1}{x} dx = 1.622 \ln |x| + C$$

$$\begin{aligned} (c) \int \frac{3x^3 - 5x^2 + 2x - 4}{x^2} dx &= \int \left(\frac{3x^3}{x^2} - \frac{5x^2}{x^2} + \frac{2x}{x^2} - \frac{4}{x^2} \right) dx \\ &= \int \left(3x - 5 + \frac{2}{x} - \frac{4}{x^2} \right) dx \\ &= \frac{3}{2}x^2 - 5x + 2 \ln |x| + \frac{4}{x} + C \end{aligned}$$