Activity 5.5 – Derivatives and Antiderivatives of Exponentials and Logarithms

1. (a) $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$; (b) $\frac{d}{dx}(\log_b(x)) = \frac{1}{(\ln b)x}$; (c) $\frac{d}{dx}(\ln(2x-10)) = \frac{2}{2x-10} = \frac{1}{x-5}$

   (d) $\frac{d}{dx}\left(\log_{10}\left(3x^2+9x+7\right)\right) = \frac{6x+9}{(\ln 10)\left(3x^2+9x+7\right)}$; (e) $\frac{d}{dt}\left((\ln(2t+1))^3\right) = 3(\ln(2t+1))^2 \cdot \frac{2}{2t+1}$

   (f) $\frac{d}{du}\left(4u^2 \ln(u)\right) = (8u)(\ln(u)) + \left(4u^2\right)\left(\frac{1}{u}\right)$

2. (a) $y' = e^x$; (b) $y' = (\ln b)\cdot b^x$; (c) $f'(x) = (\ln 5)\cdot 5^x$

   (d) $A'(t) = 1000\cdot\ln(1.03)\cdot(1.03)^{12t}\cdot 12$; (e) $g'(x) = \frac{\left((\ln 10)\cdot 10^x\right)(3x-1)-\left(10^x\right)(3)}{(3x-1)^2}$

3. (a) $\int\left(5^x - e^{5x}\right)dx = \frac{5^x}{\ln 5} - \frac{1}{5}e^{5x} + C$

   (b) $\int \frac{1-3x+4x^2}{x^2}dx = \int\left(\frac{1}{x^2} - \frac{3}{x} + 4\right)dx = -\frac{1}{x} - 3\ln|x| + 4x + C$

4. (a) $\lim\limits_{x\to+\infty}\frac{\ln(7x-8)}{\ln(4x+2)} \overset{LR}{=} \lim\limits_{x\to+\infty}\frac{\left(\frac{7}{7x-8}\right)}{\left(\frac{4}{4x+2}\right)} = \lim\limits_{x\to+\infty}\frac{7}{7x-8}\cdot\frac{4x+2}{4} = \lim\limits_{x\to+\infty}\frac{28x+14}{28x-32} = \frac{28}{28} = 1$

   (b) $\lim\limits_{x\to 3}\frac{\ln(x^2-8)}{x-3} \overset{LR}{=} \lim\limits_{x\to 3}\frac{\left(\frac{2x}{x^2-8}\right)}{1} = 6$

5. (a) $k(x) = 2^2$ is a constant function. Its derivative is $k'(x) = 0$.

   (b) $g(x) = x^2$ is a power function. Its derivative is $g'(x) = 2x$.

   (c) $h(x) = 2^x$ is an exponential function. Its derivative is $h'(x) = \ln 2\cdot 2^x$.

6. (a) If $y = x^x$, then $\ln y = \ln x^x = x\ln x$, and $\frac{d}{dx}(\ln y) = \frac{d}{dx}(x\ln x)$. Therefore, $\frac{y'}{y} = \ln x + 1$

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   and $y' = y(\ln x + 1) = x^x(\ln x + 1)$.

   (b) If $y = x^n$, then $\ln y = \ln x^n = n\ln x$, and $\frac{d}{dx}(\ln y) = \frac{d}{dx}(n\ln x)$. Therefore, $\frac{y'}{y} = \frac{n}{x}$,

   and $y' = y\left(\frac{n}{x}\right) = x^n\left(\frac{n}{x}\right) = nx^{n-1}$.

   (c) $\frac{d}{dx}(\ln y) = \frac{d}{dx}\left(3\ln x + \frac{1}{2}\ln(4x-11) - 5\ln\left(1+x^2\right)\right)$ yields $y' = \frac{x^3\sqrt{4x-11}}{(1+x^2)^5}\left(\frac{3}{x} + \frac{2}{4x-11} - \frac{10x}{1+x^2}\right)$.

7. $\Delta H = \int_{T_1}^{T_2} C_P(T)\,dT = \int_{290}^{1000}\left(a + bT + \frac{c}{T^2}\right)dT$

   $= \left(46.90T + \frac{31.51\times10^{-3}}{2}T^2 - \frac{-10.08\times10^5}{T}\right)\Bigg|_{290}^{1000} = 45261.142 \text{ J/mol}$

   $\Delta S = \int_{T_1}^{T_2}\frac{C_P(T)}{T}dT = \int_{290}^{1000}\left(\frac{a}{T} + b + \frac{c}{T^3}\right)dT$

   $= \left(46.90\ln|T| + 31.51\times10^{-3}T - \frac{-10.08\times10^5}{2T^2}\right)\Bigg|_{290}^{1000} = 74.940 \text{ J/mol-K}$