Activity 5.5[1‡] – Derivatives and Antiderivatives of Exponentials and Logarithms

FOR DISCUSSION: *State the derivative formulas for the natural and base b log functions.*
State the derivative formulas for the exponential functions in base e and b.
State the integral formulas for the exponential functions in base e and b.
State the complete power rule for integration.

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1. Compute the following.

   (a) $\frac{d}{dx}\left(\ln(x)\right) =$

   (b) $\frac{d}{dx}\left(\log_b(x)\right) =$

   (c) $\frac{d}{dx}\left(\ln(2x-10)\right) =$

   (d) $\frac{d}{dx}\left(\log_{10}\left(3x^2+9x+7\right)\right) =$

   (e) $\frac{d}{dt}\left(\left(\ln(2t+1)\right)^3\right) =$

   (f) $\frac{d}{du}\left(4u^2 \ln(u)\right) =$

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[1] This activity contains new content.
‡ This activity has supplemental exercises.

2. Compute the derivative of each function.

(a) $y = e^x$

(b) $y = b^x$

(c) $f(x) = 5^x$

(d) $A(t) = 1000(1.03)^{12t}$

(e) $g(x) = \dfrac{10^x}{3x-1}$

3. Evaluate each integral.

(a) $\int \left(5^x - e^{5x}\right) dx$

(b) $\int \dfrac{1-3x+4x^2}{x^2} dx$ (**HINT**: Split up the integrand into three fractions.)

4. Compute each limit.

(a) $\lim\limits_{x \to +\infty} \dfrac{\ln(7x-8)}{\ln(4x+2)} =$

(b) $\lim\limits_{x \to 3} \dfrac{\ln(x^2-8)}{x-3} =$

5. When computing a derivative, we typically use a differentiation rule that corresponds to the type of the given function. For example, consider the following functions:

$$k(x) = 2^2 \qquad g(x) = x^2 \qquad h(x) = 2^x \qquad f(x) = x^x$$

The first three functions are already familiar to us. For each one, write down its function type and derivative.

(a) $k(x) = 2^2$ is a ________________ function. Its derivative is $k'(x) =$ ________________.

(b) $g(x) = x^2$ is a ________________ function. Its derivative is $g'(x) =$ ________________.

(c) $h(x) = 2^x$ is an ________________ function. Its derivative is $h'(x) =$ ________________.

The function $f(x) = x^x$ has a form that we have not yet encountered, namely, a variable base raised to a variable power. In the next problem, we consider a new technique that may simplify the computations needed in finding derivatives of complicated products, quotients, and powers.

6. (**OPTIONAL**) The following method of differentiation is called **logarithmic differentiation**:

- Given $y = f(x)$, apply 'ln' to both sides to get $\ln y = \ln f(x)$.
- Expand $\ln f(x)$ using the properties of logarithms.
- Differentiate implicitly to get $\frac{y'}{y}$.
- Solve for y' and replace any y's on the right with $f(x)$.

(a) Compute the derivative of $y = x^x$ using logarithmic differentiation.

(b) Let n be any real number and let $y = x^n$. Use logarithmic differentiation to prove the power rule for differentiation, $y' = nx^{n-1}$.

(c) In Examples 5.4, we expanded $y = \frac{x^3\sqrt{4x-11}}{(1+x^2)^5}$ using properties of logarithms to get

$$\ln y = 3\ln x + \tfrac{1}{2}\ln(4x-11) - 5\ln(1+x^2)$$

Use this result and logarithmic differentiation to find y'.

7. (**OPTIONAL**) In thermodynamics, *enthalpy* (H, in J/mol) is the total energy of a system, *entropy* (S, in J/mol-K) measures how evenly energy is distributed in a system, and the *constant pressure molar heat capacity* ($C_P(T)$, in J/mol-K) is the heat required to change the temperature of a substance by T Kelvin. For a given substance, there exist constants a, b, and c such that

$$C_P(T) = a + bT + \frac{c}{T^2} \quad \text{J/mol-K}$$

For silica, $a = 46.90$, $b = 31.51\times10^{-3}$, and $c = -10.08\times10^5$. Find the net changes in enthalpy and entropy (as defined below) of silica as T increases from $T_1 = 290$ to $T_2 = 1000$.

$$\Delta H = \int_{T_1}^{T_2} C_P(T)\,dT = \int_{290}^{1000}\left(a + bT + \frac{c}{T^2}\right)dT =$$

$$\Delta S = \int_{T_1}^{T_2} \frac{C_P(T)}{T}\,dT = \int_{290}^{1000}\left(\frac{a}{T} + b + \frac{c}{T^3}\right)dT =$$