Lesson 5.4 – Logarithmic Functions

For $b > 0$ and $b \neq 1$, the graph of $y = b^x$ passes the horizontal line test, so the function $y = b^x$ is invertible. The inverse of $y = b^x$ is called the

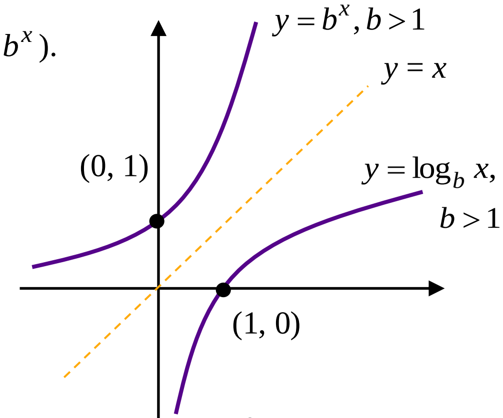
Logarithm base *b*: $y = \log_b x$ (the power of b needed to produce x)

When b is Euler's number e, we write $y = \ln x$ instead of $y = \log_e x$ and call this the

Natural logarithm: $y = \ln x$ (the power of e needed to produce x)

In science and engineering, base 10 is "common," so we call this logarithm the

Common logarithm: $y = \log_{10} x$ (the power of 10 needed to produce x)



Domain: The domain of $y = \log_b x$ is $(0, \infty)$ (the range of $y = b^x$).

Graph: Continuous on its domain; increasing and concave down for $b > 1$; decreasing and concave up for $0 < b < 1$.

***y*-intercept:** Set $x = 0$ and solve for y, but since $\log_b 0$ is undefined, there is no y-intercept.

***x*-intercept:** Set $y = \log_b x = 0$ and solve for x. This means $x = b^0 = 1$, so the x-intercept is $x = 1$.

Vertical asymptote: The horizontal asymptote of $y = b^x$ is the line $y = 0$. Therefore, the vertical asymptote of $y = \log_b x$ is the line $x = 0$ (the y-axis).

Inverse and algebraic properties: The inverse properties show that exponentials and logarithms "undo" each other, and the algebraic ones tell how logarithms affect arithmetic. We explicitly list these properties in terms of base e since it is our preferred base. Assume $x, y > 0$.

$$\log_b(b^x) = x \quad \text{and} \quad b^{\log_b x} = x \qquad \ln(e^x) = x \quad \text{and} \quad e^{\ln x} = x$$

$$\log_b(x \cdot y) = \log_b x + \log_b y \qquad \ln(x \cdot y) = \ln x + \ln y$$

$$\log_b(x / y) = \log_b x - \log_b y \qquad \ln(x / y) = \ln x - \ln y$$

$$\log_b(x^y) = y \cdot \log_b x \qquad \ln(x^y) = y \cdot \ln x$$

$$\log_b(\sqrt[y]{x}) = \frac{\log_b x}{y} \qquad \ln(\sqrt[y]{x}) = \frac{\ln x}{y}$$

Sometimes it is necessary to convert exponentials and logarithms from one base to another. We show the general conversion from base b to base a, as well the conversion from base b to base e.

Exponential change of base: $b^x = a^{(\log_a b)x}$ $\qquad b^x = e^{(\ln b)x}$

Logarithmic change of base: $\log_b x = \frac{\log_a x}{\log_a b}$ $\qquad \log_b x = \frac{\ln x}{\ln b}$