Activity 5.4 – Logarithmic Functions

1. (a) $x = \frac{1}{2}\ln(10)$

   (b) $x = e^5 - 3$

2. (a) $\ln\left(\dfrac{(x+2)\cdot\sqrt{x}}{\sqrt[3]{x^2+4}}\right) = \ln(x+2) + \frac{1}{2}\ln x - \frac{1}{3}\ln\left(x^2+4\right)$

   (b) $\log_{10}(x+1)^2 = \log_{10}(4x)$, so $(x+1)^2 = 4x$. Solve this quadratic equation to get $x = 1$.

3. (a) Set $36.5b^7 = 351.8$ to get $b = \left(\dfrac{351.8}{36.5}\right)^{1/7} \approx 1.3822$

   (b) Set $36.5(1.3822)^t = 73$ to get $t = \dfrac{\ln(2)}{\ln(1.3822)} \approx 2.14$ years

4. Set $400e^{1590k} = \frac{1}{2}\cdot 400$ to get $k = \dfrac{\ln\left(1/2\right)}{1590}$. Therefore, $A(t) = 400e^{\left(\frac{\ln(1/2)}{1590}\right)t}$, and so

   $A(2000) = 400e^{\left(\frac{\ln(1/2)}{1590}\right)(2000)} \approx 167$ mg.

5. (a) $+\infty$

   (b) $-\infty$

   (c) 0

   (d) 0

6. (a) If $T(60) = 75 = 55 + (80-55)e^{-k(60)}$, then $25e^{-60k} = 20$, and $k = -\frac{1}{60}\ln\left(\frac{20}{25}\right) \approx 0.003719$.

   (b) Since $55 + (80-55)e^{-0.003719t} = 98.6$, we have $t = -\frac{1}{0.003719}\ln\left(\frac{43.6}{25}\right) \approx -149.55$. This is 149.55 minutes *before* 11:00 p.m. Therefore, the time of death was around 8:30 p.m.