Activity 5.4† – Logarithmic Functions

FOR DISCUSSION: *The function* $\log_b x$ *is the inverse of which function?*
Describe some of the properties of logarithms.
When might we need to use a logarithmic or exponential change of base?

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1. Recall, $b^x = y$ is equivalent to $x = \log_b y$.

   (a) Solve the equation $e^{2x} = 10$ for *x*.

   (b) Solve the equation $\ln(x+3) = 5$ for *x*.

2. (a) Use the properties of logarithms to expand $\ln\left(\frac{(x+2)\cdot\sqrt{x}}{\sqrt[3]{x^2+4}}\right)$ as in Example 5.4.1.

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† This activity is referenced in Lesson 7.5

(a) Use the properties of logarithms to solve the equation $2\log_{10}(x+1) = \log_{10} x + \log_{10} 4$.

3. The internet is linked by a global network of host computers. The number of host computers was about 36.5 million in July 1998 and about 351.8 million in July 2005. We want to find a continuous exponential model of the form $N(t) = N_0 b^t$, where N is the number of host computers in millions, N_0 is the number of host computers in July of 1998, and t is the number of years since July 1998.

   (a) We are given that $N_0 = 36.5$ and $N(7) = 351.8$. Starting with $N(t) = 36.5b^t$, plug in $t = 7$ to find b. Round to four decimal places.

   (b) The **doubling time** is the time it takes for the number of host computers to grow to twice the initial amount. Find the doubling time of N. (Hint: Set $N(t) = 2N_0$ and solve for t.)

4. Let $A(t) = A_0 e^{kt}$ denote the amount of a radioactive substance remaining after t years. The **half-life** of the substance is the time it takes for the substance to decay to half of its initial amount. This is the time t such that $A(t) = 0.5A_0$. The half-life of Radium-226 is $t = 1590$ years. If a sample initially contains 400 mg, how many milligrams will remain after 2000 years? (Hint: You must find k first.)

5. Evaluate each limit.

(a) $\lim_{x \to +\infty} \ln(x) =$

(b) $\lim_{x \to 0^+} \ln(x) =$

(c) $\lim_{x \to +\infty} \dfrac{9}{\ln(x+2)} =$

(d) $\lim_{x \to 1} \dfrac{\ln(3x-2)}{2x+5} =$

6. In Lesson 7.5, we will discuss Newton's Law of Cooling and determine that the temperature T of an object at time t can be found by the formula $T(t) = T_a + (T_0 - T_a)e^{-kt}$, where T_a is the constant temperature of the surrounding medium, T_0 is the initial temperature of the object, and k is a positive constant that depends on T_a and T_0.

Suppose a coroner arrives at a crime scene to examine a corpse found in a basement that maintains a constant temperature of T_a = 55°F. At 11:00 p.m. (t = 0) , the coroner measured the body temperature to be T_0 = 80°F, and at midnight (t = 60 min) she measured it to be 75°F. Assume that the victim's body temperature was 98.6°F at the time of death.

(a) Use the formula $T(t) = T_a + (T_0 - T_a)e^{-kt}$ to find the exact value for k. Then round to six decimal places.

Begin with

$$T(60) = 75 =$$

(b) What was the time of death to the nearest minute?
(**Hint**: We want to find the time t so that $T_a + (T_0 - T_a)e^{-kt} = 98.6$. Keep in mind that t = 0 corresponds to 11:00 p.m.)