Activity 5.3 – Implicit Differentiation and Inverse Functions

1. (a) $y' = -\dfrac{x}{y}$

   (b) $y' = -\dfrac{3y}{2x}$

   (c) $y' = \dfrac{-3x^2 - y^2}{2xy - 4}$ or $\dfrac{3x^2 + y^2}{4 - 2xy}$

   (d) $y' = -\dfrac{\sqrt{y}}{\sqrt{x}}$

   (e) $y' = \dfrac{5 - ye^{xy}}{xe^{xy}}$

2. $y' = \dfrac{2 - 2x}{2y - 8}$;

   Horizontal tangents: Set $2 - 2x = 0$ to get $x = 1$. Substitute $x = 1$ into the original equation to get a quadratic equation in y with solutions $y = 0$ and $y = 8$. The points at which the circle has horizontal tangents are (1, 0) and (1, 8).
   Vertical tangents: Set $2y - 8 = 0$ to get $y = 4$. Substitute $y = 4$ into the original equation to get a quadratic equation in x with solutions $x = -3$ and $x = 5$. The points at which the circle has vertical tangents are (–3, 4) and (5, 4).

3. (a) $f^{-1}(x) = \dfrac{x+1}{6}$

   (b) $g^{-1}(x) = x^2 - 9$

   (c) $h^{-1}(x) = \dfrac{4x}{x-2}$

4. Since $f'(x) = 2x$, $\dfrac{d}{dx}\left(\sqrt{x}\right) = \dfrac{1}{2(f^{-1}(x))} = \dfrac{1}{2\left(\sqrt{x}\right)}$.