Activity 5.3[‡] – Implicit Differentiation and Inverse Functions

FOR DISCUSSION: *Give examples of explicitly and implicitly defined functions.*
Describe the technique of implicit differentiation.
How can we obtain the graph of the inverse of an invertible function?
How can we obtain the formula of the inverse of an invertible function?

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1. Use implicit differentiation to find y'.

(a) $x^2 + y^2 = 100$

(b) $x^3y^2 = 8$

(c) $xy^2 + x^3 = 4y$

[‡] This activity has supplemental exercises.

(d) $\sqrt{x}+\sqrt{y}=5$

(e) $e^{xy}=5x$

2. The equation $x^2+y^2-2x-8y+1=0$ represents a circle centered at (1, 4). Find the points on the circle at which there exist horizontal tangent lines and the points at which there exist vertical tangent lines. (**HINT**: y' is a quotient. When is a quotient zero? When is it undefined?)

3. Find an inverse formula for each function.

(a) $f(x) = 6x - 1$

(b) $g(x) = \sqrt{x+9}$

(c) $h(x) = \dfrac{2x}{x-4}$

4. (**OPTIONAL**) If $f(x) = x^2$ and $x \geq 0$, then f has an inverse, namely $f^{-1}(x) = \sqrt{x}$. Use the derivative-of-an-inverse rule to verify that $\dfrac{d}{dx}\left(\sqrt{x}\right) = \dfrac{1}{2\sqrt{x}}$.

$\left(\text{Recall, } \dfrac{d}{dx}\left(f^{-1}(x)\right) = \dfrac{1}{f'(f^{-1}(x))} \right)$.