Examples 5.2 – Derivative and Antiderivative of e^x

1. Find the derivative of $f(x) = 3e^{x^2-4x}$.

   Solution: By the chain rule, $\frac{d}{dx}\left(e^{g(x)}\right) = e^{g(x)} \cdot g'(x)$. Therefore, $f'(x) = 3e^{x^2-4x} \cdot (2x-4)$.

2. The number of CDs sold by a music store monthly is $N(p) = 6250\left(e^{-0.074p}\right)$, where p is the price in dollars per CD. The revenue R is given by the price times the number sold at that price. That is, $R(p) = pN(p) = 6250p\left(e^{-0.074p}\right)$ dollars. At what price should the store sell CDs to maximize revenue? What is the maximum revenue?

   Solution: Since revenue is a product, we must use the product rule to find its rate of change:

$$\begin{aligned} R'(p) &= (6250p)'\left(e^{-0.074p}\right) + (6250p)\left(e^{-0.074p}\right)' \\ &= (6250)\left(e^{-0.074p}\right) + (6250p)\left(e^{-0.074p}\right)(-0.074) \\ &= 6250e^{-0.074p}(1-0.074p) \end{aligned}$$

   The maximum revenue occurs when $R'(p) = 6250e^{-0.074p}(1-0.074p) = 0$. The fact that the exponential function is never zero allows us to divide through by the factor $6250e^{-0.074p}$. It follows that $1 - 0.074p = 0$, hence $p = 1/0.074$. This means that the store should sell CDs at a price of \$13.51 in order to maximize revenue, and the revenue at this price is

$$R\left(\tfrac{1}{0.074}\right) = 6250 \cdot \tfrac{1}{0.074} \cdot \left(e^{-0.074 \cdot \frac{1}{0.074}} \right) \approx \$31{,}070.90$$

3. Evaluate the integrals.

   (a) $\int\left(10e^x - 9x^2\right)dx$ (b) $\int e^{2x}dx$ (c) $\int 7e^{-t}dt$ (d) $\int 1.332e^{9\theta}d\theta$

   Solution: (a) $\int\left(10e^x - 9x^2\right)dx = 10e^x - \frac{9}{3}x^3 + C = 10e^x - 3x^3 + C$

   (b) $\int e^{2x}dx = \frac{1}{2}e^{2x} + C$

   (c) $\int 7e^{-t}dt = \frac{7}{-1}e^{-t} + C = -7e^{-t} + C$

   (d) $\int 1.332e^{9\theta}d\theta = \frac{1.332}{9}e^{9\theta} + C = 0.148e^{9\theta} + C$