Activity 5.2 – Derivative and Antiderivative of e^x

1. (a) $\frac{d}{dx}\left(5e^{4x}\right) = 5e^{4x}\cdot 4 = 20e^{4x}$

   (b) $\frac{d}{dt}\left(e^{2t-t^3}\right) = e^{2t-t^3}\cdot\left(2-3t^2\right)$

   (c) $\frac{d}{du}\left(e^u\left(1+2u^2-6u^4\right)\right) = \left(e^u\right)\left(1+2u^2-6u^4\right)+\left(e^u\right)\left(4u-24u^3\right) = e^u\left(1+4u+2u^2-24u^3-6u^4\right)$

   (d) $\frac{d}{dx}\left(\frac{x^2}{9e^{2x}+1}\right) = \frac{(2x)\left(9e^{2x}+1\right)-\left(x^2\right)\left(18e^{2x}\right)}{\left(9e^{2x}+1\right)^2}$

   (e) $\frac{d}{dx}\left(\frac{10e^x}{x^2+4e^{2x}}\right) = \frac{\left(10e^x\right)\left(x^2+4e^{2x}\right)-\left(10e^x\right)\left(2x+8e^{2x}\right)}{\left(x^2+4e^{2x}\right)^2}$

2. Set $f'(x) = (2x)\left(e^{6-x^2}\right)+\left(x^2\right)\left(e^{6-x^2}\cdot(-2x)\right) = e^{6-x^2}\left(2x-2x^3\right) = 0$ to get $x = -1, 0, 1$.

3. (a) $\int\left(5x-e^x\right)dx = \frac{5}{2}x^2-e^x+C$

   (b) $\int 5e^{-2x}dx = -\frac{5}{2}e^{-2x}+C$

   (c) $\int_0^1 e^{-t}dt = \left(-e^{-t}\right)\Big|_0^1 = \left(-e^{-1}\right)-\left(-e^0\right) = -\frac{1}{e}+1$

4. (a) $+\infty$; (b) 0; (c) 0; (d) $+\infty$; (e) 0; (f) –5;

   (g) $\lim\limits_{x\to+\infty}\frac{10e^{2x}}{x+4e^{2x}} \overset{LR}{=} \lim\limits_{x\to+\infty}\frac{20e^{2x}}{1+8e^{2x}} \overset{LR}{=} \lim\limits_{x\to+\infty}\frac{40e^{2x}}{16e^{2x}} = \frac{5}{2}$

5. Set $D'(t) = (-2)\left(e^{-5t}\right)+(-2t)\left(-5e^{-5t}\right) = e^{-5t}(-2+10t) = 0$ to get $t = 1/5$ s.

6. (c) $U(t) = 0.1496\,(1.1289)^t$ million users, where t is months since 12/31/2003

   (d) $U(48) = 50.5$ million users

   (e) $U'(t) = 0.1496\cdot e^{0.1213t}\cdot 0.1213 = 0.0181\cdot e^{0.1213t}$ million users per month, where t is months since 12/31/2003

   (f) $U'(48) = 6.11$ million users per month