Activity 5.2‡ – Derivative and Antiderivative of e^x

FOR DISCUSSION: *State the derivative and antiderivative of e^x.*
State the derivative and antiderivative of e^{kx}.

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1. Compute each of the following. Remember to use the product, quotient, and chain rules where necessary. Do not simplify your answers.

(a) $\dfrac{d}{dx}\left(5e^{4x}\right) =$

(b) $\dfrac{d}{dt}\left(e^{2t-t^3}\right) =$

(c) $\dfrac{d}{du}\left(e^u\left(1+2u^2-6u^4\right)\right) =$

(d) $\dfrac{d}{dx}\left(\dfrac{x^2}{9e^{2x}+1}\right) =$

(e) $\dfrac{d}{dx}\left(\dfrac{10e^x}{x^2+4e^{2x}}\right) =$

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‡ This activity has supplemental exercises.

2. Determine the critical points of $f(x) = x^2 e^{6-x^2}$.

3. Compute each of the following integrals.

(a) $\int \left(5x - e^x\right) dx =$

(b) $\int 5e^{-2x} dx =$

(c) $\int_0^1 e^{-t} dt =$

4. Compute the following limits. Use L'Hôpital's where necessary.

(a) $\lim\limits_{x \to +\infty} e^{2x} =$

(b) $\lim\limits_{x \to -\infty} e^{3x} =$

(c) $\lim\limits_{x \to +\infty} e^{-5x} =$

(d) $\lim\limits_{x \to -\infty} e^{-x} =$

(e) $\lim\limits_{x \to +\infty} \dfrac{9}{e^{2x} - 1} =$

(f) $\lim\limits_{x \to -\infty} \dfrac{5}{e^{3x} - 1} =$

(g) $\lim\limits_{x \to +\infty} \dfrac{10e^{2x}}{x + 4e^{2x}} =$

5. The vertical position from equilibrium, in meters, of a hydraulic piston t seconds after a downward force is applied and released is given by $D(t) = -2te^{-5t}$. Find the time at which the piston is at its furthest from equilibrium (that is, find the maximum or minimum).

6. **(OPTIONAL)** According to *Facebook.com*, the number of Facebook users has grown according to the table.

Months since 12/31/2003	Users (in millions)
6	0.13
12	1.0
24	5.5
40	20.0
59	140.0

(a) Follow the instructions to enter the input and output data into your calculator and view a scatter plot.

- Press **[STAT]** and choose **1:Edit**.
- Type inputs into L₁, and type outputs into L₂.
- Press **[2ND] [Y=]** to go to the STAT PLOTS menu.
- Choose **1:Plot1**, turn the plot **ON**, and press **[ENTER]**. You might want to turn off or clear any other equations.
- Press **[ZOOM]** and choose **9:ZoomStat**.

(b) Follow the instructions to write an exponential regression model with units, and graph the model over the data. Round the constants to four decimal places.

- Press **[STAT] [▶]** to go to the CALC menu. Press **[0]** to choose **ExpReg**.
- Depending on your calculator model, do one of the following:
  - Press **[2ND] [1] [,] [2ND] [2] [,] [VARS] [▶] [1] [1]**. You should see **ExpReg L₁,L₂,Y₁** on the home screen, OR
  - Fill in Xlist (press **[2ND] [1] [,]**), Ylist (press **[2ND] [2] [,]**), and Store RegEQ (press **[VARS] [▶] [1] [1]**).
- Press **[ENTER]**. The model will be stored in the graph menu under Y₁. Press **[GRAPH]** to view the graph of the model over the scatter plot.

(c) Call your model $U(t)$, and write it down with units. It is stored as Y₁ in the graph menu. Round the constants to four decimal places. (Your model will *not* be base *e*.)

$$U(t) =$$

(d) Use your model to estimate the number of Facebook users by the end of 2007 ($t = 48$).

$$U(48) =$$

(e) We will see later that your model from Part 1 can be written in the natural base *e* as

$$U(t) = 0.1496 \cdot e^{0.1213t} \text{ million users,}$$

where *t* is months since 12/31/2003, $6 \leq t \leq 59$. Use this model to find the rate of change model. Round the constants to four decimal places, and don't forget units.

(f) How quickly was the number of Facebook users increasing by the end of 2007?

$$U'(48) =$$