Examples 5.1 – Exponential Growth and Decay

1. In 1950, both Lineville and Powertown had populations of 1000 people. The population of Lineville was increasing by a constant 50 people per year, while the population of Powertown was increasing by a constant 5% per year. Write models for these populations, and then view their graphs on the same set of axes in the window [0, 20] × [1000, 2650]

   Solution: We set up a table that shows the growth of each population, and use it to deduce the formulas.

Years after 1950	Population of Lineville	Population of Powertown
0	1000	1000
1	$1000 + 50\cdot 1 = 1050$	$1000(1.05)^1 = 1050$
2	$1000 + 50\cdot 2 = 1100$	$1000(1.05)^2 = 1102$
…	…	…
t	$1000 + 50t$	$1000(1.05)^t$

   For instance, in 1980 the population of Lineville was $100 + 50\cdot 30 = 2500$ people, while the population of Powertown was $1000(1.05)^{30} = 4321$ people.

2. Write a discrete model for each situation.

   (a) Colony A begins with 250 bacteria and grows by 11% per day.

   (b) Colony B begins with 675 bacteria and declines by 9% per day.

   Solution: (a) Colony A has $A(t) = 250(1+0.11)^t = 250(1.11)^t$ bacteria after t days.

   (b) Colony B has $B(t) = 675(1-0.09)^t = 675(0.91)^t$ bacteria after t days.

3. Which is the better deal: 6.25% annual interest compounded monthly, or 6.20% annual interest compounded continuously? (Advertised rates are usually called **nominal rates**.)

   Solution: We write a model for each option and compute the annual rate after the effects of compounding (i.e., the **effective rate**):

   $$\text{Option 1: } y_1(t) = A\left(1+\tfrac{0.0625}{12}\right)^{12t} = A\left(\left(1+\tfrac{0.0625}{12}\right)^{12}\right)^t = A(1.\underbrace{0643}_{\textbf{6.43\% per year}})^t \quad \text{(rounded)}$$

   $$\text{Option 2: } y_2(t) = Ae^{0.062t} = A\left(e^{0.062}\right)^t = A(1.\underbrace{0640}_{\textbf{6.40\% per year}})^t \quad \text{(rounded)}$$

   Even though Option 2 offers continuous compounding, Option 1 is still the better deal.