Activity 5.1 – Exponential Growth and Decay

1. (a) If $M(t) = at + b$, then $M(0) = b = 500$, $M(2) = 2a + 500 = 245$, and $a = –127.5$.
   The model is $M(t) = –127.5t + 500$ mg, where t is hours after the injection.
   (b) If $M(t) = A(1 – r)^t$, then $M(0) = A = 500$, $M(2) = 500(1 – r)^2 = 245$, and $1 – r = 0.7$.
   The model is $M(t) = 500(0.7)^t$ mg, where t is hours after the injection.

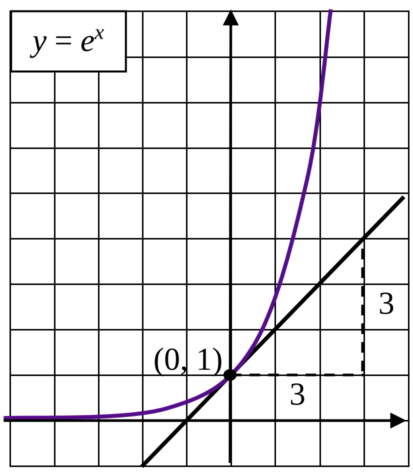
2. (a) $m(0) = 90$ grams
   (b) $m(40) \approx 27$ grams
   (c) decay rate of 0.03 = 3%

3. $P(t) = 77.2e^{0.016t}$ million tons, where t is years since 2004

4.

n compounding per yr	$\left(1+\frac{1}{n}\right)^n$ dollars after 1 yr
1 (yearly)	$\left(1+\frac{1}{1}\right)^1 = \$2.00000000$
12 (monthly)	$\left(1+\frac{1}{12}\right)^{12} = \$2.61303529$
365 (daily)	\$2.71456748
525,600 (every minute)	\$2.71827922
↓	↓
$+\infty$	\$2.718281828459…

5.



METHOD 1: $f'(0) = \frac{d}{dx}\left(e^x\right)\Big|_{x=0} = \frac{RISE}{RUN} = \frac{3}{3} = 1$

METHOD 2: $f'(0) = \frac{d}{dx}\left(e^x\right)\Big|_{x=0} = \lim_{\Delta x \to 0} \frac{e^{\Delta x} - 1}{\Delta x} = 1$

Δx	–0.1	–0.01	–0.001	→	0	←	0.001	0.01	0.1
$\frac{e^{\Delta x}-1}{\Delta x}$	0.9516	0.9950	0.9995	→	1	←	1.0005	1.0050	1.0517

6. $\lim_{n\to+\infty}\left(1+\frac{r}{n}\right)^n = \lim_{mr\to+\infty}\left(1+\frac{r}{mr}\right)^{mr} = \lim_{m\to+\infty}\left(1+\frac{1}{m}\right)^{mr} = \lim_{m\to+\infty}\left(\left(1+\frac{1}{m}\right)^m\right)^r = \left(\lim_{m\to+\infty}\left(1+\frac{1}{m}\right)^m\right)^r$;

   The limit inside the outer parentheses is equivalent to $\lim_{n\to+\infty}\left(1+\frac{1}{n}\right)^n = e$, hence

$$\lim_{n\to+\infty}\left(1+\frac{r}{n}\right)^n = \left(\lim_{m\to+\infty}\left(1+\frac{1}{m}\right)^m\right)^r = (e)^r = e^r$$