Examples 4.3 – Continuity and L'Hôpital's Rule

1. (a) Give an example of a function that is continuous but not differentiable.
   (b) Give an example of a function that is differentiable but not continuous.

   Solution: (a) An example of a continuous and non-differentiable function is $f(x) = |x|$.

   On one hand, $\lim_{x\to 0^-} f(x) =$ _____ and $\lim_{x\to 0^+} f(x) =$ _____,

   which shows that f is continuous at $x = 0$.

   On the other hand, $\lim_{x\to 0^-} f'(x) =$ _____ and $\lim_{x\to 0^+} f'(x) =$ _____,

   which shows that f is not differentiable at $x = 0$.

   (b)

2. Find constants c and d that make the piecewise function f continuous everywhere.

$$f(x) = \begin{cases} 2 - x\ , & x < 1 \\ cx^2 + d\ , & 1 \le x < 2 \\ \frac{1}{2}x + 2\ , & x \ge 2 \end{cases}$$

   Solution:

3. Use L'Hôpital's Rule to find $\lim_{x\to 2} \frac{x^2 - x - 2}{3x^2 - 9x + 6}$ and $\lim_{x\to +\infty} \frac{4x^2 - 2x + 7}{3x^2 + 9x - 1}$.

   Solution: