Activity 4.3 – Continuity and L'Hôpital's Rule

1. (a) $\displaystyle \lim_{x\to 1} \frac{x^2-6x+5}{x^3-1} \overset{LR}{=} \lim_{x\to 1} \frac{2x-6}{3x^2} = -\frac{4}{3}$

(0/0)

(b) $\displaystyle \lim_{x\to -2} \frac{x^3+3x^2-4}{x^3+4x^2+4x} \overset{LR}{=} \lim_{x\to -2} \frac{3x^2+6x}{3x^2+8x+4} \overset{LR}{=} \lim_{x\to -2} \frac{6x+6}{6x+8} = \frac{3}{2}$

(0/0) (0/0)

(c) $\displaystyle \lim_{x\to 3} \frac{-2x^3+5x}{x-3}$ has the form –39/0, so L'Hôpital's Rule does not apply here. The limit is either $+\infty$, $-\infty$, or DNE (vert. asymp. at $x = 3$). We must check left- and right-hand limits: $\displaystyle \lim_{x\to 3^-} \frac{-2x^3+5x}{x-3} = \frac{(-)}{(-)}\infty = +\infty$; $\displaystyle \lim_{x\to 3^+} \frac{-2x^3+5x}{x-3} = \frac{(-)}{(+)}\infty = -\infty$; Therefore, $\displaystyle \lim_{x\to 3} \frac{-2x^3+5x}{x-3}$ DNE.

(d) $\displaystyle \lim_{x\to 0} \frac{3x^9+4x^5-x^3}{x^7-10x^2+4} = \frac{0}{4} = 0$

(e) $\displaystyle \lim_{x\to 4} \frac{5x-20}{x^3-3x^2-4x} \overset{LR}{=} \lim_{x\to 4} \frac{5}{3x^2-6x-4} = \frac{1}{4}$

(0/0)

(f) $\displaystyle \lim_{x\to -1} \frac{x^3+3x^2+3x+1}{x^3+x^2-x-1} \overset{LR}{=} \lim_{x\to -1} \frac{3x^2+6x+3}{3x^2+2x-1} \overset{LR}{=} \lim_{x\to -1} \frac{6x+6}{6x+2} = \frac{0}{-4} = 0$

(0/0) (0/0)

2. (a) $\displaystyle \lim_{x\to +\infty} \frac{11x^2-4x+7}{15x^2+2} \overset{LR}{=} \lim_{x\to +\infty} \frac{22x-4}{30x} \overset{LR}{=} \lim_{x\to +\infty} \frac{22}{30} = \frac{11}{15}$

$(+\infty/+\infty)$ $(+\infty/+\infty)$

(c) (i) $\displaystyle \lim_{x\to +\infty} \frac{11x^{97}-4x^{33}+x^2-10}{15x^{97}+22x^{54}+6x^{12}-2} = \lim_{x\to +\infty} \frac{11x^{97}}{15x^{97}} = \frac{11}{15}$

(ii) $\displaystyle \lim_{x\to +\infty} \frac{4x^5+3x^3+x^2}{-x^5+8x-7} = \lim_{x\to +\infty} \frac{4x^5}{-x^5} = -4$

3. (a) (i) $\displaystyle \lim_{x\to -\infty} \frac{6x^2+4x}{5x^3-3x^2-2} \overset{LR}{=} \lim_{x\to -\infty} \frac{12x+4}{15x^2-6x} \overset{LR}{=} \lim_{x\to -\infty} \frac{12}{30x-6} = 0$

$(+\infty/-\infty)$ $(-\infty/+\infty)$ $(12/-\infty)$

(ii) $\displaystyle \lim_{x\to -\infty} \frac{9x^3-4x^2+3x-2}{x^2+3x} \overset{LR}{=} \lim_{x\to -\infty} \frac{27x^2-8x+3}{2x+3} \overset{LR}{=} \lim_{x\to -\infty} \frac{54x-8}{2} = -\infty$

$(-\infty/+\infty)$ $(+\infty/-\infty)$ $(-\infty/2)$

(b) (i) $\displaystyle \lim_{x\to +\infty} \frac{6x^{50}-7x^9+3x^5-10x^2}{5x^{71}+21x^{33}-66x^{12}+1} = \lim_{x\to +\infty} \frac{6x^{50}}{5x^{71}} = \lim_{x\to +\infty} \frac{6}{5x^{21}} = 0$

(ii) $\displaystyle \lim_{x\to -\infty} \frac{9x^{45}-10x^{17}+2x^{10}-10x^2+4}{x^{39}-21x^{30}+40x^{15}+x^8-9x} = \lim_{x\to -\infty} \frac{9x^{45}}{x^{39}} = \lim_{x\to -\infty} \frac{9x^6}{1} = +\infty$

4. Set $c(2)^2 + 7(2) = (2)^3 - c(2)$ to get $4c + 14 = 8 - 2c$. Therefore, $c = -1$.