Activity 4.3[1†‡] – Continuity and L'Hôpital's Rule

FOR DISCUSSION: *What are the three conditions required for continuity at a point x = a?*
Under what conditions can L'Hôpital's Rule be used?
What is the difference between L'Hôpital's Rule and the quotient rule?

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1. Evaluate each limit. Use L'Hôpital's Rule whenever possible.

(a) $\lim_{x \to 1} \frac{x^2 - 6x + 5}{x^3 - 1} =$

(b) $\lim_{x \to -2} \frac{x^3 + 3x^2 - 4}{x^3 + 4x^2 + 4x} =$

(c) $\lim_{x \to 3} \frac{-2x^3 + 5x}{x - 3} =$

(d) $\lim_{x \to 0} \frac{3x^9 + 4x^5 - x^3}{x^7 - 10x^2 + 4} =$

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[1] This activity contains new content.
[†] This activity is referenced in Lesson 4.2.
[‡] This activity has supplemental exercises.

(e) $\lim_{x \to 4} \dfrac{5x-20}{x^3-3x^2-4x} =$

(f) $\lim_{x \to -1} \dfrac{x^3+3x^2+3x+1}{x^3+x^2-x-1} =$

In Lesson 4.2, we used a shortcut to find the horizontal asymptotes of a rational function by only considering the leading terms in the numerator and denominator. Let us investigate this further.

2. (a) Use L'Hôpital's Rule as many times as necessary to find the limit. Simplify your answer.

$$\lim_{x \to +\infty} \frac{11x^2-4x+7}{15x^2+2} =$$

(b) Notice what happened to all of the lower-power terms in the numerator and denominator by the time you finished Part (a). Circle only the terms that mattered.

(a) Circle only the terms that matter in each limit. Then evaluate the limit without using L'Hôpital's Rule.

(i) $\lim_{x \to +\infty} \dfrac{11x^{97}-4x^{33}+x^2-10}{15x^{97}+22x^{54}+6x^{12}-2} =$

(ii) $\lim\limits_{x\to+\infty} \dfrac{4x^5+3x^3+x^2}{-x^5+8x-7} =$

3. (a) Use L'Hôpital's Rule as many times as necessary to find each limit.

(i) $\lim\limits_{x\to-\infty} \dfrac{6x^2+4x}{5x^3-3x^2-2} =$

(ii) $\lim\limits_{x\to-\infty} \dfrac{9x^3-4x^2+3x-2}{x^2+3x} =$

(b) Notice that in Part 3(a), the numerator or denominator of lower degree is reduced to a constant, while the one of higher degree still contains at least one term with an *x*. In other words, the limit was either $0/\pm\infty = 0$, or $\pm\infty/0 = \pm\infty$. Circle only the terms that matter in each limit. Then evaluate the limit without using L'Hôpital's Rule.

(i) $\lim\limits_{x\to+\infty} \dfrac{6x^{50}-7x^9+3x^5-10x^2}{5x^{71}+21x^{33}-66x^{12}+1} =$

(ii) $\lim\limits_{x\to-\infty} \dfrac{9x^{45}-10x^{17}+2x^{10}-10x^2+4}{x^{39}-21x^{30}+40x^{15}+x^8-9x} =$

SUMMARY: Let $\frac{N(x)}{D(x)}$ be a rational function. Suppose that ax^n is the leading coefficient of $N(x)$ and that bx^d is the leading coefficient of $D(x)$.

If $n < d$, then $\lim\limits_{x \to \pm\infty} \frac{N(x)}{D(x)} = 0$ ($y = 0$ is a horizontal asymptote in both directions)

If $n > d$, then $\lim\limits_{x \to \pm\infty} \frac{N(x)}{D(x)} = +\infty$ or $\lim\limits_{x \to \pm\infty} \frac{N(x)}{D(x)} = -\infty$ (no horizontal asymptotes)

If $n = d$, then $\lim\limits_{x \to \pm\infty} \frac{N(x)}{D(x)} = \frac{a}{b}$ ($y = a/b$ is a horizontal asymptote in both directions)

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4. Find the constant c that makes the function f continuous everywhere.

$$f(x) = \begin{cases} cx^2 + 7x, & x < 2 \\ x^3 - cx, & x \geq 2 \end{cases}$$